Risk-Shifting, Fuzzy Capital Constraint, and the Risk-Taking Channel of Low interest Rates[*†]

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Abstract: We construct a model where financial asset overpricing due to risk shifting can be moderated by capital requirements. Imperfect information about the level of capital per unit of risk, however, introduces uncertainty about the risk exposure of intermediaries. Overestimation of the level of capital of financial intermediaries, or the extent of regulatory arbitrage, may induce households to infer that higher asset prices are due to a decline of risk. This mechanism can explain the low risk premia paid by US financial intermediaries between 2000 and 2007 in spite of their increasing exposure to risk. Moreover, the underestimation of risk is larger the lower the level of the risk-free interest rate.

JEL Codes: G14, G21, E52

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1 Introduction

The goal of this paper is to explain why, in the run up to the sub-prime crisis, US financial intermediaries were able to pay non-increasing risk premia[1] while their leverage increased[2]. To do so, we introduce the capital of financial intermediaries into the risk shifting model developed by Allen and Gale (2000). In their model, households can invest in risky assets only indirectly, by lending to financial intermediaries. Households require a risk premium on this loan because they anticipate that financial intermediaries will default in the bad state of the world. However, the limited liability of intermediaries implies that they will take too much risk. A bubble results, in the sense that the price of the risky asset will be higher than in the case in which households, who do not have limited liabilities, can directly invest in the risky asset.

In our model, the amount of risk taken by households through lending to intermediaries, i. e. how much and the interest rate at which they lend, will crucially depend on the level of capital held by intermediaries. Households, who do not observe the risk of the risky assets *ex ante*, try to infer the level of risk from the price of the risky asset and their assumption on the level of capital held by financial intermediaries.

Our contribution is twofold. First, we show that the patterns of risk premia and leverage ratios observed in the US between 2000 and 2007 can be understood only if investors underestimated the intermediaries' incentives to take risks. We show how investors that form rational expectations may wrongly deduce from rising asset prices that the aggregate risk is decreasing, and thus charge a low risk premium on their loans to intermediaries. This will be the case when they underestimate the degree of regulatory arbitrage of intermediaries in the form of eroding the capital they pledge on risky assets.As argued by Acharya and Schnabl (2009) and Rochet (2008), one of the reasons why the exact extent of the regulatory capital ratio can be opaque is that intermediaries use off-balance sheet conduits to "play" the level of capital. Uncertainty about the level of capital requirement then implies uncertainty about the extent of risk associated with assets held by banks. We call *fuzzy capital constraint* the uncertainty surrounding the actual capital requirement, due to regulatory arbitrage. We prove that households under-

[1] Comparing CDS across sector allows controlling for a general trend in risk premia. For instance a "saving glut" may increase the demand for all assets and decrease risk premia on all assets. Nevertheless, comparing CDS across sectors reveals the percieved risk of a given sector compared to the rest of the economy (see Section 2).

[2] Acharya *et al.* (2009), Blanchard (2008), Brender and Pisani (2010), Brunnermeier (2008), Greenlaw *et al.* (2008).

estimate the risk of the risky assets if they overestimate the level of capital of intermediaries. The model can therefore replicate one of the most puzzling stylized facts of the banking crisis: risk premia did not increase because the depletion of capital that financial intermediaries effectively pledged to their riskiest investments was underestimated by uninformed investors (be they households, pension funds, regulators or even managers of the largest banks). In a broader perspective, capital position are likely to be opaque during periods of major financial innovation or deregulation.

Second, the model points to a risk taking channel of low real interest rates : the misperception of risk is larger at lower levels of interest rates. This is because the effect of the interest rate on asset prices is larger when the leverage of financial intermediaries is high. As a consequence, lower interest rates imply a larger effect of changes in the level of capital on the price of risky assets, and in turn on the perception of risk by investors[3]. We discuss in the paper how this channel translates into a risk-taking channel of monetary policy.

Related literature.

This article focuses on the link between the leverage of financial intermediaries, asset prices and interest rates. It relates to the results of Adrian and Shin (2010) and Geanakoplos (2009), who have highlighted the effects of financial intermediaries' leverage on asset prices. It also provides a theoretical underpinning for the empirical results of Ioanidou, Ongena and Peydro (2008), Maddaloni, Peydró and Scopel (2008), Ciccarelli, Maddaloni and Peydro (2009), Altunbas, Gambacorta and Marques-Ibanez (2010), Adrian and Shin (2010) and Shin (2009) who showed that accommodative monetary policy stance are associated with more risk taking by banks. We hence provide a theory for what Borio and Zhu (2009) and Adrian and Shin (2010) call the "risk-taking" channel of monetary policy.

In the risk-shifting literature, our paper relates first to Allen and Gale (2000)'s contribution, where they showed how limited liability on the part of debt issuers leads to over-investment in risky assets. Barlevy (2008) proved that risk-shifting also implies bubbles within more general frameworks of financial intermediation (i.e. when the formation of financial contracts is endogenous); he also generalized risk-shifting to a continuous-time dynamic framework. Challe and Ragot (2011) expand the risk-shifting model to the case in which the supply of loans is

[3] In fact, the model we use is real, and the interest rate is also real, rather than nominal. We assume that monetary policy can affect, possibly only temporarily, the level of the real interest rate on the storage asset. We discuss other factors such as the low level of interest rates in Section 4.

endogenous[4].

Finally, two other papers more recent than this one developed models on similar issues. Dell'Aricia, Laeven and Marquez (2010) developed another model of the risk taking channel of monetary policy framed in a moral hazard set up for banks'capital. Challe, Mojon and Ragot (2012) show that the proportion of banks that prefer a risky investment portfolio over a diversified, less risky, one decreases with the level of interest rates.

The paper proceeds as follows. Section 1 documents the stylized facts about the crisis. Section 2 presents the model. Section 3 solves the model with symmetric information. Section 4 presents the results with asymmetric information. Section 5 concludes. All figures are gathered in section 6. Section 7 is the appendix.

2 Stylized facts on the pre-subprime crisis

We dig out three major stylized facts of the period that preceded the sub-prime crisis: the US banking sector increased its exposure to credit risk and liquidity risk, the perceived riskiness of US financial intermediaries did not increase, and the effective level of bank's capital was difficult to assess during the period.

1. Risk-taking in the US banking sector

It is now consensual that US financial intermediaries increased their risk exposure during the decade leading up to the crisis. This took the form of increased leverage on the part of US investment banks. For instance, the Security and Exchange Commission (henceforth, SEC) reports that, between 2003 and 2007, the mean leverage ratio (defined as the ratio between overall debt and bank's equity) of the 5 major investment banks[5] jumped from 22 to 30. And among these five investment banks, only one survived the crisis as a stand alone institution.

[4] It is also important to underline the difference between the risk-shifting literature and the literature on endogenous credit constraints. The latter analyses how asymmetric information introduces external finance premiums and collateral constraints. This literature accounts well for the financial accelerator, either in the boom phase, when the rising price of collateral releases credit constraints (Kiyotaki and Moore, 1997) or in the bust phase, when the collapse in asset prices tightens the credit constraint considerably (Holmstrom and Tirole, 1997). However, these models face some difficulties in explaining why there are equilibria with too much credit and overinvestment in the risky asset.

[5] i.e. Lehman Brothers, Bear Stern, Merril Lynch, Goldman Sachs and Morgan Stanley.

Moreover, according to Fed Flow of Funds figures, the ratio between US households' debt and US nominal GDP rose from 70% to 98% between end-2000 and end-2007. This increase in US private debt coincided with an expansion of the size of US banks' balance sheet: the ratio between the debt of the US commercial banking sector and nominal GDP rose from 59% to 76% between 1999Q4 and 2007Q4.

This expansion in the size of the banks' balance sheets was accompanied by an increase in "off-balance sheet leverage"., as documented in Acharya and Schnabl (2009). This allowed financial intermediaries to generate profits without the need of additional funding in compensation for their commitment to potentially bear future losses: in fact, the unit of risk born by each dollar of the US banking system's equity had increased markedly.

2. The perceived riskiness of financial intermediaries were stable

US banks, however, benefited from very low level of risk premia paid on their debt. A look at the 10-year interest rate spread between US banks corporate and US government bonds (Fig. 1) shows, for instance, that the premia paid on the risk of banks' default had been non-increasing from 2000 to mid 2007: the price of credit risk for banks had even declined somewhat between 2002 and 2007.

The evolution of banks' expected default frequencies (henceforth, EDFs) is another indicator of the easiness for banks to have access to market funding during 2002-2007.The worldwide decreasing trend which occurred in banks' EDFs between 2002 and 2007 (see Fig. 2) suggests either that market investors were considering lower probabilities of banks' defaults, or that they were requiring lower risk premia to invest in banks' capital. The same evidence arises from the comparison on CDS contracts on US banks intermediaries, and on issuers from others economic sectors.

To sum up, we observe that, during the years leading up to the crisis, the US banking sector experienced very favorable funding conditions, and faced very low risk premia on its debt, while at the same time increasing its leverage (for investment banks) or, more generally, its exposure to credit risk and liquidity risk *via* off-balance sheet vehicles. We also notice that the increase in banks' assets was concentrated on assets which require very low capital funding. These products, considered as quite safe by regulatory standards, were however at the root of significant losses for banks, be it for credit or liquidity risk, after the beginning of the financial crisis.

3. The evolution of capital requirements

Several factors explain why capital requirements and capital norms were complacent during this period. Most of all, one of the very purpose of Basel II was to authorize banks to economize on their capital base through the use of internal models of risks. Blundell-Wignall and Atkinson (2008) and Rochet (2008) highlight the difficulty for outsiders to extract extensive information on the extent of risk undertaken by financial intermediaries .Such complexity can only have favoured interpretations and implementation of capitalization that would align with the vested interest of the industry. Finally, the accounting rules concerning the consolidation of off-balance sheet entities were singled out by the Financial Stability Forum for creating "a belief that risk did not lie with arrangers and led market participants to underestimate firms' risk exposures" (April 2008).[6]

3 The model

There are two dates $t = 1, 2$. The economy comprises four types of agents: households, financial intermediaries, entrepreneurs and initial sellers. We first describe the asset structure available to these agents and then their decision program.

3.1 Assets

Agents make their investment choices at date 1 and get assets returns at date 2. Four financial assets are available in the economy:

1. A storage asset F, which has a constant return τ. This asset is available in infinite supply.

2. A safe asset whose supply X_S is variable, and whose return is r_S. The safe asset will be issued by entrepreneurs who have access to an iso-elastic production function $f(.) = X^{1-\eta}/(1-\eta), \eta < 1$.

3. A risky asset in fixed supply X_R, which return is R^*. R^* equals R with probability π and 0 with probability $(1-\pi)$,which is the level of "*economic risk*" in the model. The price of the risky asset in period 1 is denoted as P. The assumption of fixed supply simplifies the model and is considered as the benchmark case. It is relaxed in Section 7.1, as a robustness check.

[6] This question is actually one of the point in the agenda of the G20 and similar concerns about off-balance sheet vehicules has been brought up by academics (see Acharya and Schnabl 2009), official regulators and central bankers (see for instance speeches of C. Noyer and B. Bernanke in 2008)

We make moreover the following technical assumption, which relates the concavity of the production function η and the extent of economic risk $1-\pi$.

$$\eta > \frac{1-\pi}{\pi} \tag{1}$$

This assumption, which insures uniqueness of the equilibrium, is satisfied for reasonable values of the parameters.[7]

4. The debt B issued by financial intermediaries and acquired by households.

Financial assets in this economy can be interpreted in a number of ways:

- The storage asset may for instance represent deposit facilities at the central bank, or cash. Indeed, it allows agents to invest without limit at a low and constant rate. In what follows, we will use the return on the storage asset as a proxy for the interest rate set by the monetary policy authority.
- The safe asset accounts for investment grade bonds. It can be interpreted as a loan to the "real" sector in order to finance investment or production.
- Finally, the risky asset encompasses all types of investments whose expected returns are higher than the return on bonds. It can be either real estate mortgages, junk bonds or stocks.

3.2 Agents

3.2.1 Financial intermediaries

There is a unit mass of financial intermediaries (which we henceforth also designate as "banks"), who are risk-neutral, and who receive an endowment W^f at the beginning of date 1. Agents maximize their consumption over the two periods with a discount factor β, such that:

$$\beta < 1/\tau \tag{2}$$

[7] In equilibrium we will have $f'(X_S) = r_S$ or $X_S = r_S^{-\frac{1}{\eta}}$. If η is too small, the volume (and the share) of safe assets held by financial intermediaries is quite sensitive to interest rates. As a consequence, the riskiness and the ex ante return of the entire portfolio could decrease when r_S decreases. The ex ante return on the whole portfolio could increase when r_S decreases, which would generate multiple equilibria. (1) is a sufficient condition relating the curvature η and the risk π which ensures that it is not the case.

This assumption implies that the intermediaries are comparatively impatient, so that they want to borrow in period 1. In addition, they enjoy a private benefit U from being intermediaries. This benefit guarantees that these agents accept to operate as intermediaries rather than consuming all their endowment in period 1. They thus seek to maximize $c_1^f + \beta E\left[c_2^f\right] + U$, where c_1^f and c_2^f are the period 1 and period 2 consumption levels. Financial markets open in period 1 after goods market. As a consequence, financial intermediaries can bring to financial markets as equity only the value K of the wealth which is not consumed in period 1:

$$c_1^f \leq W^f - K. \tag{3}$$

Financial intermediaries can invest in all existing assets. They won't invest in the storage asset because they have access to the safe asset, which yields a higher return. Thus, their balance sheet is composed of a risky asset, PX_R, and a safe asset, X_S, on the asset side, whereas their liabilities are either equity, K, or debt, B. The amount K stands for the fraction of resources invested by the intermediaries themselves in their business. The resource constraint of financial intermediary is

$$PX_R + X_S = B + K. \tag{4}$$

We assume that banks are subject to a market norm of "risk coverage" or "capital requirements" by either financial regulation or market discipline[8]. Banks have to invest from their endowment at least Δ per unit value of the risky asset:

$$K \geq \Delta PX_R \tag{3}$$

Following Allen and Gale (2000), we assume that financial intermediaries raise some funds using debt contracts, and that lenders are not able to fully observe the investment decisions of financial intermediaries. Lenders cannot discriminate between banks because these are identical *ex ante*. Hence, they will demand the same interest rate r whatever the size of the loan they grant to the financial intermediary.

Financial intermediaries can default on their debt. Default occurs when the intermediary's wealth is negative in period 2. Their second period consumption c_2^f is thus

$$c_2^f \leq \max\{R^* X_R + r_S X_S - rB, 0\} \tag{5}$$

[8] Indeed, the level of capital requirement need not exclusively be the one set by regulators. It can also be the market norm on the acceptable level of capital for a given level of risk-taking.

Financial intermediaries therefore choose their debt level B, and the composition of their portfolio (X_S, X_R) to solve the following program

$$\max_{K,B,X_R,X_S} c_1^f + \beta E\left[c_2^f\right] + U$$

subject to the conditions (3)-(5).

3.2.2 Households

Finally, there is a unit mass of households, who are risk-neutral. Each household j receives an endowment W_j^H at the beginning of date 1. To simplify the algebra, and without any loss of generality, households maximize their date 2 consumption only.

As in Allen and Gale (2000), we introduce the following form of market segmentation. Households cannot directly invest in the risky asset or in the safe asset, and they can only either invest in the storage asset, or lend to financial intermediaries an amount B at the interest rate r. This assumption captures the advanced skills and accumulated rents (asset management abilities, private information, and so on) needed to trade corporate bonds and, in general, sophisticated financial products.

Households do not know the level of risk in the economy, summarized by π. Moreover, they can't observe the structure of the composition of banks' liabilities, K and B. This assumption reflects the inability of each household to know from their own loans the extent of banks' aggregate indebtedness. As argued above, the rationale for this assumption is the complexity of the bank liability structure due to off-balance sheet liabilities. Technically, households know their portfolio but they do not observe the portfolio of other households and we do not restrict our analysis to symmetric equilibria.

The key assumption of our model is that households do not know the true value of capital requirement Δ, but they have a belief Δ^H, which may differ form Δ. We study the various cases below. These assumptions are meant to reflect that households, investors, or even rating agencies have a hard time assessing the extent of risk effectively born by financial intermediaries, due to regulatory arbitrage.

Each household j chooses the composition of their financial portfolio in order to maximize his final consumption, taking market prices, his expectations of aggregate risk and the capital ratio of banks as given:

$$\max_{F,B^H} \beta E\left[c_j^H\right] \quad (6)$$

$$F_j + B_j^H \leq W_j^H \text{ (at date 1)} \quad (7)$$

$$c_j^H \leq \rho B_j^H + \tau F_j \text{ (at date 2)} \quad (8)$$

where $E[.]$ is the expectation operator and β is the discount factor. The expectations are formulated on R^* for a given value Δ^H. In the budget constraints, F_j is the amount invested in the storage asset, and B_j^H is the amount lent to intermediaries. We further denote $B^H = \int_j B_j^H$, $F = \int_j F_j$, and $c = \int_j c_j^H$ the aggregate amount of loans to intermediaries, the aggregate investment in the storage asset, and the aggregate households consumption, respectively. The stochastic interest rate ρ that all households receive ex-post on their loans to financial intermediaries is uncertain, and depends on probability of default of intermediaries. Intermediaries default in the bad state of the world, in which the return on the risky asset is 0, as the cost of the repayment of the debt will be always higher than the return on the safe asset. In case of default, households get the residual value of the banks' portfolio $r_S X_S + R^* X_R$ for the realized value of $R^* = 0$ (as default only occurs in the bad state), so that ρ in case of default is $r_S X_S / B$. When intermediaries do not default, households get the return r.

3.2.3 Entrepreneurs and initial sellers

There is a unit mass of entrepreneurs who maximize period 2 consumption, denoted as c^e. They have no wealth and they need to borrow in period 1 to produce in period 2. Their production function is $f(X) = X^{1-\eta}/(1-\eta)$. They borrow an amount denoted as X_S^e from the market at a rate r_S, to maximize their period 2 consumption $c^e = f(X_S^e) - r_S X_S^e$. This maximization over X_S^e yields the simple relationship

$$r_s = f'(X_S^e) \quad (9)$$

Initial sellers are agents who sell the risky assets to intermediaries at period 1, consume and leave the economy. These agents are only introduced as a simple way of creating a supply of the risky asset at the beginning of period 1. Initial sellers have no choices to make, and simply consume in period 1 the amount obtained from the sale of the risky asset:

$$c^i = P X_R$$

3.2.4 Equilibrium

For given parameters, and a given value of households expectations Δ^H, an equilibrium of this economy is a set of prices r, r_S, P and quantities $B^H, F, c^H, K, X_S, X_S^e$, and a risk assessment by households π^H, such that 1) quantities solve the program of all agents at given prices and given households belief Δ^H, and 2) Markets clear $X_R = 1$, $X_S = X_S^e$ and $B = B^H$.

3.3 Pareto Efficient Equilibria

We first derive the set of Pareto efficient allocations. In order to do this, we maximize a general welfare function, which weights the utility of the four types of agents. This can be written, with obvious notations for the Pareto weights

$$W = \omega^H \beta E\left[c^H\right] + \omega^f \left(c_1^f + \beta E\left[c_2^f\right]\right) + \omega^i c^i + \omega^e \beta c^e \tag{10}$$

$$\text{with } \omega^H, \omega^f, \omega^i, \omega^e > 0 \text{ and } \omega^H + \omega^f + \omega^i + \omega^e = 1$$

Expectations in the objective function are only taken on the economic risk, R^*. Budget constraints are:

$$W^f + W^h = c_1^f + F + X_S + c^i \tag{11}$$

$$\tau F + f(X_S) + R X_R = c^H + c_2^f + c^e \tag{12}$$

As $\tau < 1/\beta$, forming the Lagrangian for the maximization of (10) subject to the constraints (11) and (12), one can check that the solution has the following properties

$$F = 0 \text{ and } f'(X_S) = \frac{1}{\beta}$$

The allocation of the central planner can be achieved in the decentralized economy if we remove any market segmentation and allow for lump sum transfers. In this case, households can directly lend to entrepreneurs and buy the safe asset. In this equilibrium, the interest rate on the safe asset is $1/\beta$ and the price of the risky asset is equal to its fundamental value

$$P^* = \beta \pi R$$

4 Asset Prices and Households' Belief

In this section we derive the price of the risky asset by solving the program of financial intermediaries, and provide the intuitions for the main results of the paper.

We solve the program of financial intermediary under two conjectures. The first is that the capital requirement constraint is always binding, hence $K = \Delta P X_R$. This case is of course the relevant one for this model. We derive below the condition for the constraint to be binding.

The capital norm is binding if financial intermediaries are sufficiently impatient, i.e.

$$\pi r < 1/\beta \tag{13}$$

This inequality stipulates that the expected cost of the debt πr (because the debt is repaid only outside the bad state, which occurs with probability π) must not be too high. If the expected cost of the debt is too high, intermediaries would want to invest all of their wealth to decrease their expected debt burden, and the capital norm constraint would therefore not bind. As r is determined in equilibrium, we show below that the condition (13) is fulfilled for a wide range of parameter values.

The solution of the program of intermediaries yields the equilibrium price of the risky asset:

$$P = \frac{\beta \pi R}{\Delta + \beta \pi r (1 - \Delta)} \tag{14}$$

This asset price equilibrium is the main equation of the model. First note that when there is no capital requirement ($\Delta = 0$), the price is simply $P = R/r$, which is the case studied by Allen and Gale (2000).[9] As intermediaries default in the case of a catastrophic aggregate shock, their demand for the risky asset is always higher than under the first best equilibrium. Indeed, as $\pi r < 1/\beta$, one finds $P > P^*$. Asset prices are thus too high. Second, when capital requirements increase, the price of the risky asset decreases. Taking r as given, increasing Δ implies a cost in the form of additional foregone consumption in period 1, an effect that dominates the reduction in size of the loan that needs to be repaid with probability π.

Thus, in partial equilibrium, the price of the risky asset can increase for two reasons: either because π increases, which means that the expected return of the risky asset is higher, or because Δ decreases (the amount of *ex ante* risk-shifting increases).

Maximization with respect to the demand for the safe asset X_S yields that the funding cost of financial intermediaries is equal to the return on the safe asset, as in Allen and Gale (2000):

[9] In their model, Allen and Gale show how incomplete debt contracts limit debtors' losses in the bad state of the world (losses fall on lenders). In other words, debt contracts act as call options for borrowers. This implies that borrowers only focus on the good state of the world when deciding the composition of their portfolio: the share of the portfolio at risk is higher and the price of risky assets is inflated above its level in a world without segmentation or complete contracts.

$r = r_s$. This is necessary and sufficient in order to avoid infinite riskless profit opportunities on the part of financial intermediaries, while guaranteeing a positive demand in equilibrium.

The demand for the safe asset yields

$$f'(X_S) = r_S = r \Longrightarrow X_S = \left[f'(r)\right]^{-1} \tag{15}$$

The previous equalities are valid whatever the beliefs on the part of households about the economic environment. These beliefs will however determine the interest rate charged by households. We now determine this interest rate for each of our two polar assumptions on the information available to households.

The basic assumption of the model is that households observe prices, but they do not know the riskiness π of the assets and form beliefs about this value observing prices. The price of the asset is given by (14) for the true value of Δ and π, because it results from a no-arbitrage condition for intermediaries, who know the real value π and the real Δ. Households deduce a value of π^H which is consistent with price P and their belief Δ^H. It has to be the case that

$$P = \frac{\beta\pi R}{\Delta + r\beta\pi\,(1-\Delta)} = \frac{\beta\pi^H R}{\Delta^H + r\beta\pi^H\,(1-\Delta^H)} \tag{16}$$

We deduce the following inference for households

$$\pi^H = \pi\frac{\Delta^H}{\Delta + r\beta\pi\,(\Delta^H - \Delta)} \tag{17}$$

Due to the condition $r\beta\pi < 1$, if $\Delta^H > \Delta$, then $\pi^H > \pi$, and if $\Delta^H < \Delta$, then $\pi^H < \pi$. In addition, if $\Delta^H = \Delta$ then $\pi^H = \pi$. In words, if households overestimate capital requirements, they underestimate the risk and if they underestimate capital requirements, they overestimate the risk. Moreover, when households correctly evaluate the level of capital requirements, they correctly infer the right level of aggregate risk. This case, in analyzed in the next proposition. All the proof are left in Appendix.

Proposition 1 *If $\Delta^H = \Delta$, household expectations of the aggregate risk is correct $\pi^H = \pi$ and*

$$\frac{\partial B}{\partial\Delta} < 0 \text{ and } \frac{\partial(r-\tau)}{\partial\Delta} < 0$$

The proposition implies that decreasing capital per unit of the risky asset increases both the volume of debt of intermediaries and the credit risk premium, $r - \tau$, when this decrease is anticipated by households. This result is due to two main effects. First, the overall general equilibrium effect of a decrease in Δ is an increase in the intermediaries' debt level, as financial intermediaries have a greater incentive to increase their exposure to risk by issuing debt.

Second, when households observe that Δ declines, the effect of risk-shifting on asset prices is somewhat moderated by the response of the risk premium, $r - \tau$. Investors realize that the residual value of the assets in case of default decreases. They hence request a larger default risk premium $r - \tau$ to compensate for the increased cost of default. This version of the model is therefore *not* consistent with the stylized facts of the sub-prime cycle. As showed by Figure 1, indeed, banks have been able to borrow at lower risk premia during the five years leading up to the crisis, in spite of increasing their exposure to US housing assets.

To summarize, if the risk per unit of capital pledged by financial intermediaries—which could be due, for instance, to a larger scale of off-balance sheet activities—can account for an increase in their debt level, it cannot explain the path of the risk premia between 2000 and 2007. We therefore assert that risk-shifting, *per se*, is not sufficient to replicate the stylized fact of the *subprime crisis*. Before the crisis, banks and financial intermediaries have in fact benefited from extremely favorable funding conditions, a feature which cannot arise in the presence of known changes to capital requirements.

5 Solution with under-estimation of risk

We now assume that households belief about the level of capital requirement is higher than the one actually faced by financial intermediaries: $\Delta^H > \Delta$. In this case, the amount of capital pledged by financial intermediaries is lower than what households expect. Households overestimate the probability of success of the risky asset $\pi^H > \pi$.

Households form their inference about the residual value of their portfolio, $\frac{rX_S}{B^H}$, in the following way. First, from the observation of the amount of risky asset in the economy X_R, and from their belief about Δ, households assume that the level of banks' capital is:

$$K^H = X_R \Delta^H P$$

From the balance sheet constraint of banks given by equality (**??**), households form the following expectation of the amount of debt

$$B^H = X_S + P X_R \left(1 - \Delta^H\right) \tag{18}$$

Third, the no-arbitrage condition for households must now be written according to the expectations of households. As they anticipate that the regulatory constraint will be high with probability p and low with probability $1-p$, they adjust their portfolio so that:

$$\pi^H r + \left(1-\pi^H\right)\frac{rX_S}{B^H} = \tau \tag{19}$$

Using equations (17) in order to substitute for π^H, the expressions for X_S given by (15), the value of B implied by the balance sheet constraint of the intermediary, and the fact that $r_S = r$, we obtain an equation for the equilibrium interest rate r which depends only on known parameters and functional forms.

In order to obtain analytical insight, we assume that households belief about capital requirement is not too far away from the true one, that is we assume that $\varepsilon \equiv \Delta^H - \Delta$ is small. The next proposition present some first results.

Proposition 2 *If ε is small, we have*

$$\frac{\partial \pi^H}{\partial \varepsilon} > 0, \frac{\partial\left(r-\tau\right)}{\partial \varepsilon} < 0 \text{ and } \frac{\partial B}{\partial \varepsilon} > 0$$

The previous proposition summarizes the effect of an increase the overestimation of capital requirement Δ^H (or a decrease in capital requirements Δ keeping belief Δ^H constant). Households become more optimistic about the risk of the asset π^H. They hence charge a lower risk premium, what allows financial intermediaries to borrow more. The proposition proves that unexpected regulatory arbitrage might explain the trends before the crisis. The next result concerns the interaction between the risk free rate and the risk free rate τ.

Proposition 3 *For a given $\Delta^H > \Delta$, we have*

$$\frac{\partial \pi^H}{\partial \tau} < 0, \frac{\partial\left(r-\tau\right)}{\partial \tau} > 0$$

When households overestimate actual capital requirements, a decrease in the risk free rate τ generates an exacerbation of their optimistic bias about the risky asset. The reason stems from equation (16). When the risk free rate τ decreases, the lending rate to financial intermediaries r also decreases under general conditions. Asset prices P increases, but as households have not the correct belief about Δ they attribute part of this increase to a decrease in the riskiness of the asset $\frac{\partial(r-\tau)}{\partial\varepsilon} < 0$, what contributes to increase the borrowing of financial intermediaries. As monetary policy can have an effect on real interest rate, at least in the short-run and due to sticky prices, the model can explain that monetary policy might affect aggregate risk taking[10].

The predictions of the model for risk perception are actually consistent with the empirical results produced by Altumbas, Gambacorta and Marquez (2010). They found that the Expected Default Frequencies, and other market-based measures of bank's risks as perceived by financial market participants, react positively to changes in interest rates: a lower interest rate leads investors to perceive banks as less risky. Turning to banks' risk-taking, which may be interpreted as banks exploiting their ability to borrow cheaply from financial markets, a number of recent studies, including Jimenez *et al.* (2007), Ioannidou, Ongena and Peydro (2008), and Ciccarelli, Maddaloni and Peydro (2009) show that credit standards are correlated to the level of interest rates. Lower interest rates therefore imply lower credit standards, including for customers who are perceived as presenting a higher credit risk.

It is important to stress, however, that the model bears results for the impact of the level of interest rates on risk perception and risk-taking irrespective of the source of variation in interest rates. The interest rate in the model is real, and can therefore be influenced by several factors. During the decade leading up to the crisis, several explanations had been put forth in order to explain the low level of nominal and real interest rates. According to Taylor and Williams (2009), US monetary policy had been overly accommodative. Bernanke (2010), however, stressed instead that China's excess savings had played a major role in keeping the long end of the US yield curve at comparatively low levels. Either of these factors may in turn have been amplified by the phenomenon of "search for yields", as emphasized by Rajan (2006). We don't take a stand on these alternative possible drivers of the level of the interest rates, and only stress that the endogenous mechanism described in our model would hold for either of them.

Central banks may actually seek to trigger risk taking by investors when they lower interest rates, and perhaps even more so when confronted to a liquidity trap. In such circumstances, that characterize the US, Japan, the euro area and several other advanced economies since 2009, central banks may wish to spur demand by lowering the cost of capital for many classes of borrowers, including small and medium enterprises, across the board of financing instruments, from bank loans to stocks through high yield bonds. The search for yields of investors would

[10] The model therefore provides an explanation for two complementary aspects of "the risk taking channel of monetary policy" defined by Borio and Zhu (2008) as : "the impact of changes in policy rates on either risk perceptions or risk-tolerance and hence on the degree of risk in the portfolios, on the pricing of assets, and on the price and non-price terms of the extension of funding".

ideally channel funds to positive net present value pools of projects, helping to bring demand closer to potential output.

What the model highlights, however, is that the search for yield and risk taking can in part result from the wrong inference of risks from asset prices. This is because interest rates are central in the valuation of assets and the inference on risk incentives. It points to the interdependence of monetary and capital based prudential policies in a world where risk incentives cannot be assessed with certainty.

6 The case of risk over-estimation

The previous section has focused on the case where households over estimate capital and wrongly infer the level of collapse risk to be consistent with the stylized fact on the pre-subprime crisis the previous analysis. This case may also be relevant for period of rapid financial innovations, where households may not perceive the discrepancies between capital requirements and actual risk taking by financial intermediaries.

The symmetric case of risk overestimation (and underestimation of capital constraints) may however also be interesting. The case of excess prudence, in the sense that banks are thought to be less capitalized than then are may help understand other periods of history. In a recent paper Malmendier and Nagel (2011) show that households who experienced the Great Depression are less likely to invest in stock markets or to participate to financial markets. We interpret this behavior as an underestimation of constraints imposed on banks after the Glass-Steagall act. Anticipating that both the banking system and the risky assets are more risky than what they really are, households ask for high return to compensate for the perceived risk. Empirical support for the view that investors' appetite for risks varies over time can also be found in Gilchrist and Zakrajsek (2012). There, they show large and persistent swings in the price of risk, defined as the part of bond risk premia that are not explained by "fundamentals" on the risk of default derived from stock prices using Merton's valuation of firms stocks as option to default. The price of risk has been consistently negative from 2003 to 2007. It has also remained positive for several periods including, around 2000, around 2008 and through out the 1980s. Again, the periods of overestimation of risk may be due to an underestimation of the regulatory constraints imposed on financial intermediaries.

7 Alternative specification of the model

7.1 Elastic supply of the risky asset

We have assumed that the supply of the risky asset was fixed $X_R = 1$. We now show that the results are robust to the introduction of an elastic supply. Assume now that the risky asset is produced by risky entrepreneurs, instead of behind sold in period 1 by initial sellers, as previously assumed. Risky entrepreneurs have access to a risky technology, whose risk is perfectly correlated. If they invest an amount Y in period 1 they produce an amount $g(Y) = Y^{1-\theta}/(1-\theta)$, with a probability π in period 2 and they produce an amount 0 with probability $1-\pi$. If r_R is the lending rate to risky entrepreneurs, they maximize their period 2 consumption, denoted as c^R, which is $c^R = \pi[g(Y) - r_r Y]$. It yields, $Y = r_R^{-1/\theta}$.

To simplify the algebra and economic interpretation, it is easier to consider $P = 1/r^R$, which is the price of one unit of risky asset, and $X_R = Y/P$, which is the quantity of risky assets in period 1. One unit of risky asset costs P in period 1 and has a payoff 1 with a probability π and 0 with a probability $1-\pi$ in period 2. With this change of variable the supply of risky asset in period 1 is

$$X_R = P^{\frac{1-\theta}{\theta}}$$

Risky entrepreneurs invest an amount $X_R = PY$ in period 1 and pay to lenders $r_R X_R = Y$ in period 2. Previously, we considered the special case where $\theta = 1$.

The next proposition summarizes the effect of an increase in the supply elasticity of the risky asset (which is a decrease in θ). We need an additional assumption which is $\beta\pi R > 1$ to consider the relevant case. The condition $\beta\pi R > 1$ is a sufficient condition for $P > 1$. In this case, the value of the risky asset sold in period 1 increases when the elasticity increases. Indeed, the price is high enough for risky entrepreneurs to increases their production of risky asset. We consider this case as the relevant one, which may be consistent with trends before the crisis.

Proposition 4 *If $\beta\pi R > 1$ and $\varepsilon > 0$, then when the supply of the risky asset becomes more elastic (ie. θ decreases), we find 1) the value of risky asset PX_R increases 2) the risk premium $r - \tau$ decreases 3) π^H increases,*

The proof is in Appendix. The proposition states that when the elasticity of the supply of risky asset increases (and when the value of risky asset sold in period 1 increases) and when agents overestimate the regulatory constraints, then the risk premium $r - \tau$ increases and the expectations of success becomes more optimistic. Households indeed overestimate the return of a higher quantity of asset. Financial intermediaries benefit more from this overestimation and increase their demand of the risky asset. This raises the price that households interpret as a increase in the probability of success of the risky asset. Finally, note that when $\varepsilon = 0$ (ie. $\Delta^H = \Delta$), then $\pi^H = \pi$, whatever the elasticity of the supply of the risky asset (see equation 17)

Financial innovation before the financial crisis may have increase the elasticity of the supply of the risky asset. The previous propositions states that this may have distorted anticipations if and only if households overestimated the regulatory constraint.

7.2 Regulatory risk

We simply assumed above that households may form wrong expectations about the level of regulatory requirement. It is possible to introduce an additional shock on the level of capital requirement. The shock is known by banks, but unknown to the savors. In this framework households form on average correct expectations about the average level of capital requirement. For an unexpected negative shock to this level, we would find the same results as in the current model where $\Delta^H > \Delta$. We chose a simple structure to be able to derive theoretical result in a transparent way, to identify the effects. More general information structure could also be introduced, such as learning process, it would not create additional mechanisms to the ones studied in the current paper.

7.3 Uncertain return

In the basic, version we assumed that the return in case of success R was known, but that the probability distribution of risk, π, was unknown. An alternative modeling strategy would be to consider π as known but that the return $R = R(e)$ is uncertain and affected by private actions of financial intermediaries. In this case, the main result would be preserved: If households knew capital requirements Δ, they could infer from asset prices the real return $R(e)$ and thus the private action e. Changes in capital requirements may drive changes in private actions, but this changes would be anticipated and thus reflected in risk premia. Alternatively, when capital requirement is unknown, changes in capital requirements, and thus in equilibrium prices, would be partly understood as a higher return and would thus biased the credit risk. Although it may be hard to distinguish between uncertain return and uncertain probabilities for specific assets, our modeling choice is motivated by the direct evidence of a sharp change in the expected probability of default during the crisis as related in Section 2. Our model was thus designed to explain this bias in expectations of default.

7.4 Return and risk

In our model, a change in the probability of default π affects both the mean and the variance of the return on the risk asset. As agents are risk neutral, the effect on the variance does not affect prices, but one could still want to express the model to analyze the effect of a change in the mean return keeping its variance constant. It is possible to do so by introducing an additional risk. Assume that the risky asset is equal to 0 with a probability $1 - \pi$ and to a stochastic variable $\tilde{R}$ with a probability π. $\tilde{R}$ has a mean R and is uniformly distributed in the support $[R - \delta; R + \delta]$. If the support is small enough, default will occur only when R is equal to 0. One can then jointly choose π and δ to study the effect of a change in mean which keeps the variance constant.

8 Alternative explanations of low risk premium

8.1 Expectations of bailout

The model focuses on uncertainty of capital requirements to explain the behavior of risk premium. An alternative explanation for the low level of risk premia is that investors expected to be collectively bailed out by governments and central banks. Farhi and Tirole (2012) propose a model in which financial institutions coordinate their exposure to risks in order to increase systemic risk, and therefore the likelihood that public authorities will bail them out. Whereas their model explores the issue of risk-shifting from investors to taxpayers, we focus instead on the shifting of risk from banks to bondholders.

Obviously, the expectations of bailouts may have play a role in the evolution of risk premia. In particular, the big change in expected default frequency after Lehman's Brother default in sept 2008 may be due to partly to the revision in the probability of public bailout and partly to a reassessment of the underlying risk. To our knowledge, there is no evidence which allows to disentangle clearly between the two effects. Nevertheless, some evidence suggests that the agents underestimated the risk of default. First, The expected default frequency raised in 2007, after people started to question the quality at the asset side of specific funds proposed by banks.

Second, regulators and central bankers did not mention any risk to financial stability before 2007 and often stressed the effectiveness of financial regulations. Finally, and more generally, bank analysts did not point to any bank risk before the crisis. Although not directly quantitative, these facts indicate that the underestimation of the probability of default before the crisis may be a significant contributor to the low risk premia.

8.2 General underestimation of risk

An other explanation of low risk premia would be that all agents, including financial intermediaries, underestimated the risk of default on housing assets. Again, a global underestimation of risk can not be rejected and it may be hard to claim that banks correctly anticipated the real risk on all assets. Nevertheless, some evidence again suggest that they have been an asymmetry in the informations held by financial intermediaries and by savors. The popular indication of the fact banks knew that there were taken important risk in 2007 is the quote of Chuck Prince, then Chairman of Citigroup: "When the music stops, in terms of liquidity, things will be complicated. But, as long as the music is playing, you've got to get up and dance. We are still dancing." (Financial Times,7/9/07). Moreover, recent legal pursuits against some financial intermediaries have shown that there were an asymmetry of information between financial intermediaries and savors, at least in specific cases. An other paper elaborating on the information asymmetry between financial intermediaries and savors is Shleifer and Vishny (2010). In this paper, the expectations of households are taken as given. Instead, we endogenize them through an inflation extraction problem.

9 Concluding Remarks : Can the model explain the build up of financial fragility?

In this paper we show, first, that the combination of risk-shifting and fuzzy capital requirement can explain a puzzling fact of the sub-prime crisis, i.e. that banks could increase their exposure to risk without having to pay higher risk premia on their debt.

In a situation of uncertainty with respect to regulatory constraints, and of an opaque banking system, the increase in the observed asset prices can be interpreted as a lower aggregate risk in the economy while, in fact, asset prices are driven by greater risk-taking on the part of financial intermediaries. We also showed that this model gives rise to a risk-taking channel of low interest rate: the influence of the level of the interest rate on risk perception on the part of some agents and exposure to risk on the part of others.

Our result resonates with the popular notion that financial markets participants can form wrong inference on risks. In particular, when the effectiveness of capital requirements is not observable by agents, the signal extracted from market prices is contaminated by noise coming from excessive risk-taking behavior. In our model, market forces, by themselves, are not able to lead the economy to the optimum allocation of capital, because risk incentives are not correctly understood.

We see two obvious extensions to our model. First, it is possible to endogenize the expectations of households within a dynamic setting in which households learn about the relevant parameters. Although the results of our models would still hold even if the priors of the households were far enough from the true parameters, the resulting dynamics might generate interesting patterns. Second, it would be interesting to study the political economy aspects associated with the assessment of risk within such an economy. Sellers of the assets have an incentive to underestimate the extent of economic risk, or to generate complexity in order to increase the cost of signal extraction. This should be anticipated by households, who would then look for other sources of information. For instance, we understand the current discussion about rating agencies as part of the political economy debate on the management of risk expectations in economies where intermediaries play an important role.

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10 Figures

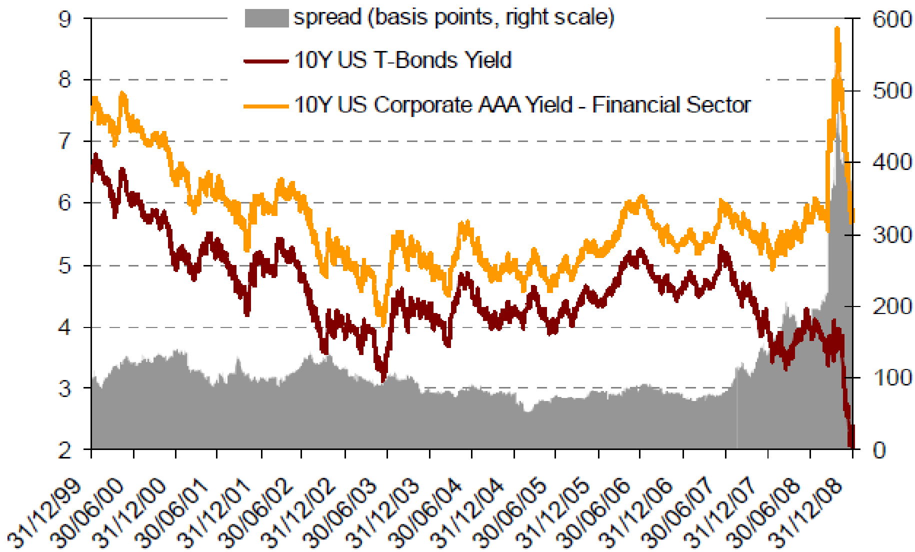

Fig 1: Spread between 10Y US T-Bonds and 10Y Bonds of US AAA Financial Companies

Source: Bloomberg

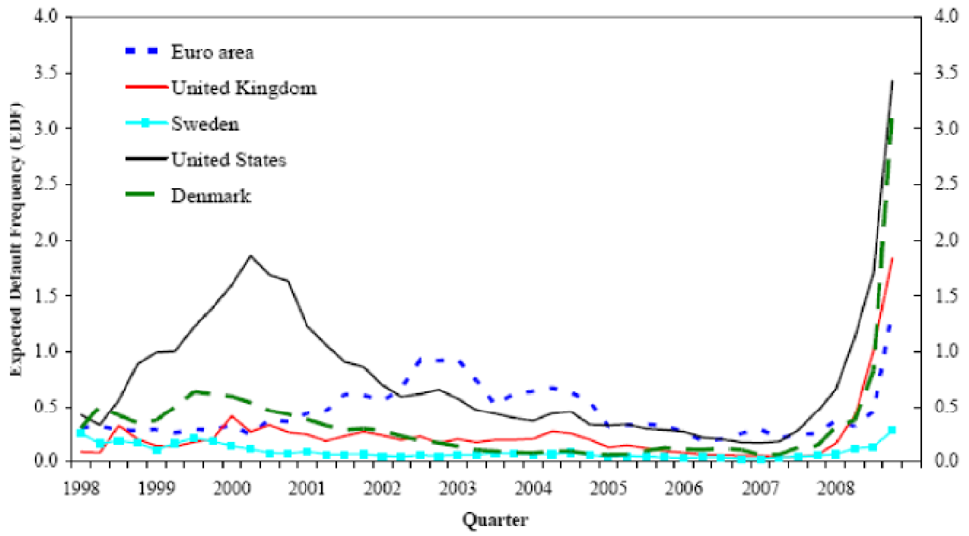

Fig 2: Expected Default Frequency of banks, over a 1-year horizon, averages by country and groups of countries

Source: Moody's KMV, from Altunbas and al. (2010)

11 Appendix

A Proof of proposition 1

Households can deduce the amount of intermediaries' capital and debt from the amount of agregate exposure to risk and with their knowledge of the coefficient Δ, $K = \Delta P X_R$. They can also infer the amount of agregate debt B from the balance sheet constraint of the intermediary.

The no-arbitrage condition for household can be written as, , with the expression for X_S given by (15) and $r = r_S$

$$\pi r + (1-\pi)\frac{r f'^{-1}(r)}{B} = \tau \tag{20}$$

Since $\pi \in]0,1[$ and $X_S < B$, the no-arbitrage condition (20) implies $r > \tau$. Equality (20) can be written as

$$B(r) = \frac{(1-\pi) r X_S}{\tau - \pi r} \tag{21}$$

We can substitute K, X_S and P by their equilibrium values given by equations (14) and (15) to obtain an expression $B(r)$:

$$B(r) = \frac{(1-\Delta) R}{\frac{\Delta}{\beta\pi} + r(1-\Delta)} X_R + f'^{-1}(r) \tag{22}$$

Let us define: $\Theta \equiv \frac{\Delta}{1-\Delta}$. Then from (15), (22) and (20), we find that the real interest rate r

satisfies the equality

$$\tau = \pi r + (1-\pi)\frac{(\Theta+\beta\pi r)\, r^{1-1/\eta}}{\beta\pi R X_R + \Theta r^{-1/\eta} + \beta\pi r^{1-1/\eta}}$$

This last equaltiy implicitely defines the interest rate by equality $M(r)=\Theta$, where

$$M(r) \equiv \beta\pi\left(\frac{(\tau-\pi r)\, r^{1/\eta}\mu X_R}{r-\tau} - r\right)$$

In the equilibrium under consideration $\pi r < \tau < 1$ and $r > \tau$. As a consequence, one can check that a sufficient condition for $M'(r) < 0$ is $\eta > \frac{1-\pi}{\pi}$, which is (1) In this case, the equality $M(r) = \Theta$ implies that r is decreasing with Δ.

From equality (20), one finds $B(r) = \frac{1-\pi}{\tau-\pi r} r^{1-\frac{1}{\eta}}$. After some algebra, one finds that $B(r)$ is increasing with r when (1) is fufilled. As a consequence, $\frac{\partial B}{\partial r} > 0$, $\frac{\partial(r-\tau)}{\partial\Delta} < 0$ and $\frac{\partial B}{\partial \Delta} < 0$.

B Proof of Proposition 2

Denote $\varepsilon = \Delta^H - \Delta$. From the equations (15), (19), (16), (17) (18) one can find that the real interest satisfies $G(r,\tau,\Delta,\varepsilon)=0$, where

$$G(r,\tau,\Delta,\varepsilon) \equiv \beta\pi\left[\frac{\left(\tau - \pi r\frac{\Delta+\varepsilon}{\Delta+r\beta\pi\varepsilon}\right) R r^{\frac{1}{\eta}}}{r-\tau} - r\left(1-\frac{\varepsilon}{1-\Delta}\right)^{-1}\right] - \frac{\Delta}{1-\Delta-\varepsilon} \qquad (23)$$

As $\tau, \Delta, \varepsilon$ are parameters of the model. The equality $G(r,\tau,\Delta,\varepsilon)=0$ pins down the equilibrium interest rate as a function of the parameters of the model. Studying the derivative of the function G, one finds the following sign for the derivatives, with obvious notations

$$G\left(\underset{-}{r}, \underset{+}{\tau}, \underset{-}{\Delta}, \underset{-}{\varepsilon}\right) = 0$$

As a consequence and by the implicit function theorem, one finds that r has the following notations $r = r\left(\underset{+}{\tau}, \underset{-}{\Delta}, \underset{-}{\varepsilon}\right)$. This proves $\frac{\partial(r-\tau)}{\partial\varepsilon} < 0$

Next, the anticipated probability can be written as $\pi^H = \pi(\varepsilon+\Delta)/(\Delta + r\beta\pi\varepsilon)$. As a consequence, one finds the following variations $\pi^H = \pi^H\left(\underset{-}{r}, \underset{+}{\varepsilon}\right)$. this proves $\frac{\partial \pi^H}{\partial\varepsilon} > 0$ (as r decreases whrn ε increases).

Finally, the budget constraint of financial intermediary together with the price (16) gives the debt leve B

$$B = r^{-\frac{1}{\eta}} + \frac{\beta\pi R}{\frac{\Delta}{1-\Delta} + r\beta\pi}$$

One easily deduce the variation $B = B\left(\underset{-}{r}\right)$ and $r = r\left(\underset{+}{\tau}, \underset{-}{\Delta}, \underset{-}{\varepsilon}\right)$. We have thus the variations $B = B\left(\underset{-}{\tau}, \underset{+}{\varepsilon}\right)$

C Proof of Proposition 3

From the proof of Proposition 2, we have $r = r\left(\underset{+}{\tau}, \underset{-}{\Delta}, \underset{-}{\varepsilon}\right)$ and $\pi^H = \pi^H\left(\underset{-}{r}, \underset{+}{\varepsilon}\right)$. As a consequence, we finds $\frac{\partial \pi^H}{\partial \tau} < 0$. The proof of the inequality $\frac{\partial (r-\tau)}{\partial \tau} > 0$ requires more algebra. This inequality is first proven for $\varepsilon = 0$. Then, a continuity argumend is invoked. From the definition 23 and the equality $G(r, \tau, \Delta, \varepsilon) = 0$, one finds

$$r - \tau = r \frac{1 - \pi \frac{\Delta+\varepsilon}{\Delta + r\beta\pi\varepsilon}}{\frac{1}{\mu_1 r^{\frac{1}{\eta}}}\left[\frac{\Delta}{1-\Delta}\frac{1}{\beta\pi} + r\right]\frac{1}{1-\frac{\varepsilon}{1-\Delta}} + 1}$$

Taking the derivative with respect to τ and setting $\varepsilon = 0$, we easily find $\frac{\partial (r-\tau)}{\partial \tau} > 0$. By continuity, this inequality is fulfilled when ε is small.

D Proof of Propositions 4

The equilibrium is given as before by the equations (15), (19), (16), (17) (18), but the supply of the risky asset is now

$$X_R = P^{\frac{1-\theta}{\theta}}$$

One can use these equations to find a single equations in r, which is

$$\pi \frac{\Delta^e}{\Delta + r\beta\pi\left(\Delta^e - \Delta\right)} r + \left(1 - \pi \frac{\Delta^e}{\Delta + r\beta\pi\left(\Delta^e - \Delta\right)}\right) \frac{1}{r^{-\frac{1}{\eta}} + \left(\frac{\beta\pi R}{\Delta + r\beta\pi(1-\Delta)}\right)^{\frac{1}{\theta}}\left(1 - \Delta^e\right)} r^{1-\frac{1}{\eta}} = \tau$$

The previous equation determines a r by the implicit function

$$\hat{G}(r, \tau, \Delta, \varepsilon, \theta) = 0$$

When $\theta = 1$, we know that

$$\hat{G}(r, \tau, \Delta, \varepsilon, 1) = G\left(\underset{-}{r}, \underset{+}{\tau}, \underset{-}{\Delta}, \underset{-}{\varepsilon}\right)$$

where G is given by (23), which corresponds to the case where $\theta = 1$.

First, when $\theta = 1$, the equilibrium real interest rate is $r_{\theta=1}$ and we know that $P(r_{\theta=1}) = \left(\frac{\beta\pi R}{\Delta + r_{\theta=1}\beta\pi(1-\Delta)}\right)^{\frac{1}{\theta}} > P^* > 1$. As a consequence, if r is in the neighborhood of $r_{\theta=1}$, we have $P(r)^{\frac{1}{\theta}}$ is decreasing with θ for a given r, and $\hat{g}(r, \tau, \Delta, \varepsilon, \theta)$ is increasing in θ for given r. Moreover, we know that we know that, when $\theta = 1$

$$\hat{G}(r, \tau, \Delta, \varepsilon, 1) = G\left(\underset{-}{r}, \underset{+}{\tau}, \underset{-}{\Delta}, \underset{-}{\varepsilon}\right)$$

As a consequence, as the function $\hat{G}$ is C^1 (i.e. one can show that the derivative are continuous in the value of the parameters, when r is in the neighborhood of its equilibrium value), we find that the equilibrium satisfies the following implicit equations, with the variations of the function $\hat{G}$ indicated below the variables.

$$\hat{G}\left(\underset{-}{r}, \underset{+}{\tau}, \underset{-}{\Delta}, \underset{-}{\varepsilon}, \underset{+}{\theta}\right) = 0$$

As a consequence, one finds r increases with θ in equilibrium. As before, we have $\pi^H = \pi^H\left(\underset{-}{r}, \underset{+}{\varepsilon}\right)$ only if $\varepsilon > 0$. As a consequence, we find that when θ decreases, r decreases and π^H increases.