# Precautionary Saving over the Business Cycle*

(Short title: Precautionary Saving over the Cycle)

Edouard Challe Xavier Ragot

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## Abstract

We study the macroeconomic implications of time-varying precautionary savings within a general equilibrium model with borrowing constraints, aggregate shocks, and uninsurable idiosyncratic unemployment risk. Our framework generates limited cross-sectional household heterogeneity as an equilibrium outcome, thereby making it possible to analyze the role of precautionary saving over the business cycle in an analytically tractable way. The time-series behavior of aggregate consumption generated by our model is closer to the data than that implied by the hand-to-mouth and representative-agent models, and it is comparable to that produced by the Krusell-Smith (1998) model.

How important are changes in precautionary asset accumulation for the propagation of business cycle shocks? In this paper, we attempt to answer this question by constructing a tractable model of time-varying precautionary saving behavior driven by countercyclical

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*Corresponding author: Edouard Challe, Ecole Polytechnique, Departement of Economics, Route de Saclay, 91128 Palaiseau, France; Email: edouard.challe@gmail.com. We are particularly grateful to three anonymous referees for their comments as well as to the editor for his guidance. We also thank Olivier Allais, Paul Beaudry, Hafedh Bouakez, Nicola Pavoni, Sergio Rebelo and Pontus Rendahl for their detailed comments at various stages of this project. We also benefited from comments by conference participants at the 2012 Hydra Workshop, the EABCN/CEPR Conference on Disaggregating the Business Cycle, the 2012 ASSA Meetings, the 2011 SED Congress, the 2011 NBER SI, the 2011 Paris Macro Finance Workshop, the 2010 Minnesota Macro Workshop, and the 2010 Philadelphia Fed Search and Matching Workshop; and by seminar participants at Ecole Polytechnique, Sciences Po, EIEF, LBS, INSEAD, HEC Paris, HEC Lausanne, ECB, Banque de France, Bank of Portugal, Université du Mans, AMSE, IIES, Tilburg University, and the University of Cambridge. Eric Mengus provided outstanding research assistance. The French Agence Nationale pour la Recherche (grant no 06JCJC0157) and chaire FDIR provided funding.

changes in unemployment risk. Because households are assumed to be imperfectly insured against this risk, they respond to such changes by altering their buffer stock of wealth. This in turn amplifies the consumption response to aggregate shocks that affect unemployment.

Our motivation for investigating the role of precautionary saving over the business cycle is based on earlier empirical evidence which points to a significant role for the precautionary motive in explaining the accumulation and variation of wealth by individuals over time. Empirical studies that focus on the cross-sectional dispersion of wealth suggest that, all else equal, households facing higher income risk accumulate more wealth or consume less, on average (see, e.g., Carroll, 1994; Carroll and Samwick, 1997, 1998; Engen and Gruber, 2001). This argument has been extended to the time-series dimension by Carroll (1992), Gourinchas and Parker (2001), Parker and Preston (2005), and, more recently, Carroll et al. (2012), who argue that changes in precautionary wealth accumulation following countercyclical changes in income volatility may substantially amplify fluctuations in aggregate consumption.[1] We construct a general equilibrium model in which the strength of the precautionary motive is explicitly related to the extent of unemployment risk, the main source of income fluctuations for most households (at least at business cycle frequencies).

Methodologically, the novelty of our approach is to propose a class of heterogeneous-agent models with incomplete markets, borrowing constraints, and both aggregate and idiosyncratic labor income shocks that can be solved under exact aggregation and rational expectations. More specifically, we outline a set of sufficient conditions about preferences and the tightness of the borrowing constraint, under which the model endogenously generates a cross-sectional distribution of wealth with a limited number of states; exact aggregation directly follows. This makes the model "tractable" in the sense that its dynamics can be summarized by a low-dimensional dynamic system, the solution to which admits a simple state-space representation. This approach makes it possible to derive analytical results and incorporate time-varying precautionary saving into general equilibrium analysis using simple solution methods–including linearization and undetermined coefficient methods. In particular, our analysis allows the derivation of a common asset-holding rule for employed households facing incomplete insurance, possibly expressed in *linear* form, which explicitly connects precautionary wealth accumulation to the risk of becoming unemployed. Additionally, our model can be simulated with several–and possibly imperfectly correlated–aggregate shocks with continuous support; we consider three such shocks in our baseline specification

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[1] In particular, Carroll et al. (2012) find the precautionary motive to be the second most important factor (after the collapse in household wealth) behind the decline in US consumption during the Great Recession.

(i.e., technology, job-finding and job-separation shocks). Thus, our approach differs from that in traditional heterogeneous-agent models with aggregate shocks a la Krusell and Smith (1998), which typically generate a full, time-varying, cross-sectional distribution of wealth that every agent must forecast in order to make their best intertemporal decisions. While we construct and simulate the simplest version of our model with limited cross-sectional heterogeneity here, we emphasize that its tractability can be exploited in many other contexts, for example when other frictions (e.g., nominal rigidities, labor market frictions, etc.) interact with incomplete insurance.[2]

In order to isolate the precautionary motive in the determination of households' savings, our general framework incorporates both patient "permanent-income" consumers and impatient consumers who are imperfectly insured and may face occasionally binding borrowing constraints. Aside from the baseline precautionary-saving case just discussed, wherein impatient households hold a time-varying buffer-stock of wealth in excess of the borrowing limit, our framework embodies two cases of special interest: the *representative-agent* model and the *hand-to-mouth* model. The representative-agent model arises in the limit of our incomplete-market model when the economy becomes entirely populated by permanent-income consumers. The hand-to-mouth model–a situation where impatient households face a binding borrowing limit in every period–endogenously arises when the precautionary motive becomes too weak to offset impatience, causing impatient households to consume their entire income in every period.[3] We link the strength of the precautionary motive–and thus whether or not impatient households are ultimately willing to save–to the deep parameters of the model, most notably the extent of unemployment risk, the generosity of the unemployment insurance scheme, and the tightness of the borrowing constraint.

We then use our framework to identify and quantify the specific role of incomplete insurance and precautionary wealth accumulation–as opposed to mere borrowing constraints for example–in determining the volatility of aggregate consumption and its co-movements with output. We thus calibrate the model to match the main features of the cross-sectional distributions of wealth and nondurable consumption in the US economy (in addition to the other usual quantities). Next, we feed the calibrated model with aggregate shocks to productivity and labor market transition rates with magnitude and joint behavior that are

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[2] See McKay and Reis (2013) or Ravn and Sterk (2013) for recent analyses of the interactions of such frictions with incomplete markets in the context of models with full cross-sectional heterogeneity.

[3] In this case, our economy collapses to a two-agent one, like those studied by Becker and Foias (1987), Kiyotaki and Moore (1997), or Iacoviello (2005), for example.

directly estimated from post-war US data. We then study their quantitative implications for a variety of aggregate and distributional statistics. We find the time-series behavior of aggregate consumption generated by our baseline precautionary-savings model to be closer to the data than those implied by the comparable hand-to-mouth and representative-agent models. To complete the picture, we also compare the moments of interest implied by our baseline precautionary-savings model with those generated by the full-fledged heterogeneous-agent model of Krusell and Smith (1998, Section IV).

Our analysis differs from earlier attempts at constructing tractable models with incomplete insurance, which typically restrict the stochastic processes for the idiosyncratic shocks in ways that makes them ill-suited for the analysis of time-varying unemployment risk. For example, Constantinides and Duffie (1996) study the asset-pricing implications of an economy in which households are hit by uninsured permanent income shocks. Heathcote et al. (2013) have generalized this approach by looking at the case where households' income is also affected by insurable transitory shocks.[4] Toche (2005), and more recently Carroll and Toche (2011), explicitly solve for households' optimal asset-holding rule in a partial-equilibrium economy where they face the risk of permanently exiting the labor market. Guerrieri and Lorenzoni (2009) analyze precautionary saving behavior in a model with trading frictions a la Lagos and Wright (2005), showing that agents' liquidity hoarding amplifies the impact of i.i.d. (aggregate and idiosyncratic) productivity shocks. Relative to these models, ours allows for stochastic transitions across labor market statuses, which implies that individual income shocks are transitory (but persistent) and have a conditional distribution that depends on the aggregate state. The model is thus fully consistent with the flow approach to the labor market and can be evaluated using direct evidence on the cyclical movements in labor market flows. Our approach is also related to Vermeylen (2006), who shows how to solve an incomplete-market model with idiosyncratic shocks by linearizing it around the steady state of its *complete-market* counterpart. In contrast, our model can be formulated nonlinearly and can accommodate aggregate shocks.

Section 1 presents the model. In Section 2, we introduce the parameter restrictions that make our model tractable by endogenously limiting the dimensionality of the cross-sectional distribution of wealth. Section 3 calibrates the model and compares its quantitative implications to the data and to alternative theoretical benchmarks. Section 4 concludes.

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[4] See also Heathcote et al. (2008) and Braun and Nakajima (2012).

# 1 The Model

The model features a closed economy with a representative firm and a continuum of households uniformly distributed along the unit interval. All households rent out labor and capital to the firm, which produces the unique (final) good in the economy. Markets are competitive but there are frictions in the financial markets, as we describe further below.

## 1.1 *Households*

Every household $i$ is endowed with one unit of labor, which is supplied inelastically to the representative firm if the household is employed. All households are subject to idiosyncratic changes in their labor market status between "employment" and "unemployment". Employed households earn a competitive market wage (net of social contributions) while unemployed households earn a fixed unemployment benefit $\delta^i > 0$.

We assume that households can be of two types, *impatient* and *patient*, distributed on the subintervals $[0, \Omega]$ and $(\Omega, 1]$ respectively, with $\Omega \in [0, 1)$. While not necessary for the construction of our equilibrium with limited cross-sectional heterogeneity, the introduction of patient households will allow us to generate a substantial degree of cross-sectional wealth dispersion since they will end up holding a large fraction of total wealth in equilibrium. The unemployment risk faced by households is summarized by two probabilities: the probability that a household employed at date $t-1$ will be unemployed at date $t$ (the job-loss probability $s_t$) and the probability that a household unemployed at date $t-1$ will remain unemployed at date $t$ (i.e., $1-f_t$, where $f_t$ is the job-finding probability). The law of motion for employment is:

$$n_t = (1 - n_{t-1}) f_t + (1 - s_t) n_{t-1}. \tag{1}$$

### 1.1.1 *Impatient households*

Impatient households maximize $\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\beta^I\right)^t u^I \left(c_t^i\right)$, $i \in [0, \Omega]$, where $c_t^i$ is (nondurable) consumption by household $i$ at date $t$, $u^I(.)$ is the period utility function satisfying $u^{I\prime}(.) > 0$ and $u^{I\prime\prime}(.) \leq 0$, and $\beta^I \in (0, 1)$ is the subjective discount factor. We restrict the set of assets that impatient households have access to in two ways. First, we assume that they cannot issue assets contingent on their employment status but only enjoy the (partial) insurance provided by the public unemployment insurance scheme. Second, we assume that these households face an (exogenous) borrowing limit in that their asset wealth cannot fall below $-\mu$, where $\mu \geq 0$. We let $e_t^i$ denote household $i$'s employment status at date $t$, with $e_t^i = 1$

if the household is employed and 0 otherwise. The budget and non-negativity constraints faced by an impatient household are:

$$a_t^i + c_t^i = e_t^i w_t^I \left(1 - \tau_t\right) + \left(1 - e_t^i\right) \delta^I + R_t a_{t-1}^i \tag{2}$$

$$c_t^i \geq 0,\ a_t^i \geq -\mu, \tag{3}$$

where $a_t^i$ represents the household's holdings of claims to the capital stock at the end of date $t$, $R_t$ is the ex-post gross return on these claims, $w_t^I$ is the real wage for impatient households, $\delta^I$ is the unemployment benefit enjoyed by these households, and $w_t^I \tau_t$ is a social contribution paid by the employed to finance the unemployment insurance scheme. The Euler condition for impatient households is:

$$u^{I\prime}\left(c_t^i\right) = \beta^I \mathbb{E}_t[u^{I\prime}(c_{t+1}^i) R_{t+1}] + \varphi_t^i, \tag{4}$$

where $\varphi_t^i$ is the Lagrange coefficient associated with the borrowing constraint $a_t^i \geq -\mu$, with $\varphi_t^i > 0$ if the constraint is binding and $\varphi_t^i = 0$ otherwise. Condition (4), together with the initial asset holdings $a_{-1}^i$, as well as the optimality conditions $\lim_{n\to\infty} \mathbb{E}_t[\beta^{It+n} a_{t+n}^i u^{I\prime}\left(c_{t+n}^i\right)] = 0$ and $\varphi_t^i\left(a_t^i + \mu\right) = 0$, fully characterize the asset holdings of impatient households.

### 1.1.2 *Patient households*

Patient households maximize $\mathbb{E}_0 \sum_{t=0}^{\infty} \left(\beta^P\right)^t u^P\left(c_t^i\right)$, $i \in (\Omega, 1]$, where $\beta^P \in \left(\beta^I, 1\right)$, $u^P(.)$ is a continuous, strictly increasing, and strictly concave function over $[0, \infty)$, and where $\sigma^P(c) \equiv -u^{P\prime\prime}(c)\, c / u^{P\prime}(c)$. Unlike impatient households, patient households have complete access to asset markets, including the full set of Arrow-Debreu securities and loan contracts.[5] Hence, patient households collectively behave like a large representative "family" of permanent-income consumers in which the family head ensures an equal marginal utility of wealth for all its members (see, e.g., Merz, 1995). Since consumption is the only argument in the period utility function, equal marginal utility of wealth implies equal consumption. Hence, we can write the budget constraint of the family as:

$$C_t^P + A_t^P = R_t A_{t-1}^P + (1 - \Omega)\left(n_t w_t^P \left(1 - \tau_t\right) + \left(1 - n_t\right) \delta^P\right), \tag{5}$$

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[5] Patient households will be more wealthy than impatient households in equilibrium –and a lot more so when we calibrate the model to match the cross-sectional distribution of wealth in the US. Under a fixed participation cost to trading Arrow-Debreu securities (as in, e.g., Mengus and Pancrazi (2012)), we expect households holding more wealth (patient households here) to be more willing to buy insurance, all else equal. Quantitatively, the results in Krusell and Smith (1998) illustrate that the behavior of *wealthy* agents facing incomplete markets and borrowing constraints is almost indistinguishable from that of fully insured agents.

where $C_t^P$ ($\geq 0$) and $A_t^P$ denote the consumption and end-of-period asset holdings of the family (both of which must be divided by $1-\Omega$ to find the per-family member analogues), and $w_t^P$ and $\delta^P$ are the real wage and unemployment benefit for patient households. The Euler condition for patient households is given by:

$$u^{P\prime}\left(C_t^P/\left(1-\Omega\right)\right) = \beta^P \mathbb{E}_t\left[u^{P\prime}\left(C_{t+1}^P/\left(1-\Omega\right)\right) R_{t+1}\right]. \tag{6}$$

This condition, the terminal condition $\lim_{n\to\infty}\mathbb{E}_t[\left(\beta^P\right)^{t+n} A_{t+n}^P u^{P\prime}(C_{t+n}^P/\left(1-\Omega\right)] = 0$ and the initial asset holdings $A_{-1}^P$ fully characterize the optimal consumption path of patient households.

## 1.2 *Production*

The representative firm produces output, $Y_t$, out of capital, $K_t$, and the units of effective labor supplied by households. We let $n_t^I$ and $n_t^P$ denote the firm's use of impatient and patient households' labor input. We then define $Y_t = z_t G\left(K_t, n_t^I + \kappa n_t^P\right)$ as the aggregate production function, where $\kappa > 0$ is the relative efficiency of patient households' labor (with the efficiency of impatient households' labor normalized to one); $\{z_t\}_{t=0}^{\infty}$ is a stochastic aggregate productivity process with mean $z^* = 1$; and where $G\left(.,.\right)$ exhibits positive, decreasing marginal products and constant returns to scale (CRS). As will become clear in Section 3, the introduction of an efficiency premium for patient households (i.e., $\kappa > 1$) raises their labor income share, which is necessary to match the empirical cross-sectional *consumption* dispersion (for any plausible level of wealth dispersion). With $k_t \equiv K_t/\left(n_t^I + \kappa n_t^P\right)$ and $g\left(k_t\right) \equiv G\left(k_t, 1\right)$, we have $Y_t = z_t\left(n_t^I + \kappa n_t^P\right) g\left(k_t\right)$. The optimality condition for firms is then given by:

$$z_t g'\left(k_t\right) = R_t - 1 + \nu, \tag{7}$$

where $\nu \in [0,1]$ is the depreciation rate. The optimal demands for the two labor types in a perfectly competitive labor market must satisfy $z_t G_2\left(K_t, n_t^I + \kappa n_t^P\right) = w_t^I = w_t^P/\kappa$, where $w_t^I$ is the real wage per unit of effective labor.

## 1.3 *Market Clearing*

By the law of large numbers and the fact that all households face identical transition rates in the labor market, the equilibrium numbers of impatient and patient households working in the representative firm are $n_t^I = \Omega n_t$ and $n_t^P = \left(1-\Omega\right) n_t$, respectively. Consequently, effective labor is $n_t^I + \kappa n_t^P = \left[\Omega + \left(1-\Omega\right)\kappa\right] n_t$ and the capital stock is $K_t =$

$[\Omega + (1-\Omega)\kappa]\, n_t k_t$. Moreover, by the CRS assumption, the price of one unit of effective labor is $w_t^I = z_t \left[ g(k_t) - k_t g'(k_t) \right]$. Now, let $F_t(\tilde{a}, e)$ denote the measure at date $t$ of impatient households with beginning-of-period asset wealth $\tilde{a}$ and employment status $e$, with $a_t(\tilde{a}, e)$ and $c_t(\tilde{a}, e)$ the corresponding policy functions for assets and consumption. Market clearing for claims to the capital stock requires that:

$$A_{t-1}^P + \Omega \sum_{e=0,1} \int_{\tilde{a}=-\mu}^{+\infty} a_{t-1}(\tilde{a}, e)\, \mathrm{d}F_{t-1}(\tilde{a}, e) = [\Omega + (1-\Omega)\kappa]\, n_t k_t, \tag{8}$$

where the left hand side is total asset holdings by all households at the end of date $t-1$ and the right hand side is the demand for capital by the representative firm at date $t$.

Clearing of the goods market requires:

$$C_t^P + \Omega \sum_{e=0,1} \int_{\tilde{a}=-\mu}^{+\infty} c_t(\tilde{a}, e)\, \mathrm{d}F_t(\tilde{a}, e) + I_t = z_t [\Omega + (1-\Omega)\kappa]\, n_t g(k_t), \tag{9}$$

where the left hand side includes the consumption of all households as well as aggregate investment, $I_t = [\Omega + (1-\Omega)\kappa]\left(n_{t+1}k_{t+1} - (1-\nu)\, n_t k_t\right)$, and the right hand side is output.

Finally, we require the unemployment insurance scheme to be balanced:

$$\tau_t n_t \left[ \Omega w_t^I + (1-\Omega)\, w_t^P \right] = (1-n_t) \left[ \Omega \delta^I + (1-\Omega)\, \delta^P \right], \tag{10}$$

where total unemployment contributions (left hand side) equal total unemployment benefits (right hand side).

**Definition 1**. An *equilibrium* is defined as sequences of i) household decisions $\{C_t^P, c_t^i, A_t^P, a_t^i\}_{t=0}^{\infty}$, ii) the firm's capital per effective labor unit $\{k_t\}_{t=0}^{\infty}$, and iii) aggregate variables $\{n_t, w_t^I, R_t, \tau_t\}_{t=0}^{\infty}$ such that conditions (4) and (6)–(10) are satisfied, given the forcing sequences $\{f_t, s_t, z_t\}_{t=0}^{\infty}$ and the initial wealth distribution $\left(A_{-1}^p, a_{-1}^i,\right)_{i\in[0,\Omega]}$.

## 2 Equilibrium with Limited Cross-Sectional Heterogeneity

Dynamic general equilibrium models with incomplete markets and borrowing constraints usually generate a cross-sectional distribution of wealth with a large number of states. This is because individual wealth is determined by one's entire history of idiosyncratic shocks (Aiyagari, 1994; Krusell and Smith, 1998). In this paper, we make specific assumptions about impatient households' period utility and the tightness of the borrowing constraint. These assumptions ensure that the cross-sectional distribution of wealth has a finite number of wealth states as an equilibrium outcome. As a result, the economy is characterized by a finite number of heterogeneous agents whose behavior can be aggregated exactly, thereby making it possible to represent the model's dynamics via a standard (small-scale) dynamic system. In the remainder of the paper, we focus on the simplest equilibrium, which involves exactly two possible wealth states for impatient households. However, we show in the technical appendix that this approach can be generalized to construct tractable equilibria with any finite number of wealth states.

## 2.1 *Assumptions and Conjectured Equilibrium*

Let us first assume that the instant utility function of impatient households, $u^I(c)$, is i) continuous, increasing, and differentiable over $[0,+\infty)$; ii) strictly concave with local relative risk aversion coefficient $\sigma^I(c) = -cu^{I\prime\prime}(c)/u^{I\prime}(c) > 0$ over $[0,c^*]$, where $c^*$ is an exogenous, positive threshold; and iii) linear with slope $\eta > 0$ over $(c^*,+\infty)$ (see Figure 1). This utility function, which is an extreme form of decreasing relative risk aversion, implies that high-consumption (i.e., relatively wealthy) impatient households do not mind moderate consumption fluctuations–as long as the implied optimal consumption level says inside $(c^*,+\infty)$–but dislike substantial consumption drops–those that would cause consumption to fall inside the $[0,c^*]$ interval.

Given this utility function, we derive our equilibrium with limited cross-sectional heterogeneity by construction; we first guess the general form of the solution and verify ex-post that the set of conditions under which the conjectured equilibrium was derived prevails in equilibrium. Our first conjecture is that an employed impatient household is sufficiently wealthy for its chosen consumption level to lie above $c^*$, while an unemployed, impatient household chooses a consumption level below $c^*$. In other words, we are constructing an equilibrium in which the following condition holds:

$$\textbf{Condition 1}: \forall i \in [0,\Omega],\ e_t^i = 1 \Rightarrow c_t^i > c^*,\ e_t^i = 0 \Rightarrow c_t^i \leq c^*. \tag{11}$$

As we shall see shortly, one implication of this utility function and of the ranking of consumption levels is that employed households fear unemployment. Consequently, they engage in *ex ante* precautionary saving behavior in order to limit (without being able to fully eliminate) the associated rise in marginal utility.

The second feature of the equilibrium we are constructing is that the borrowing constraint

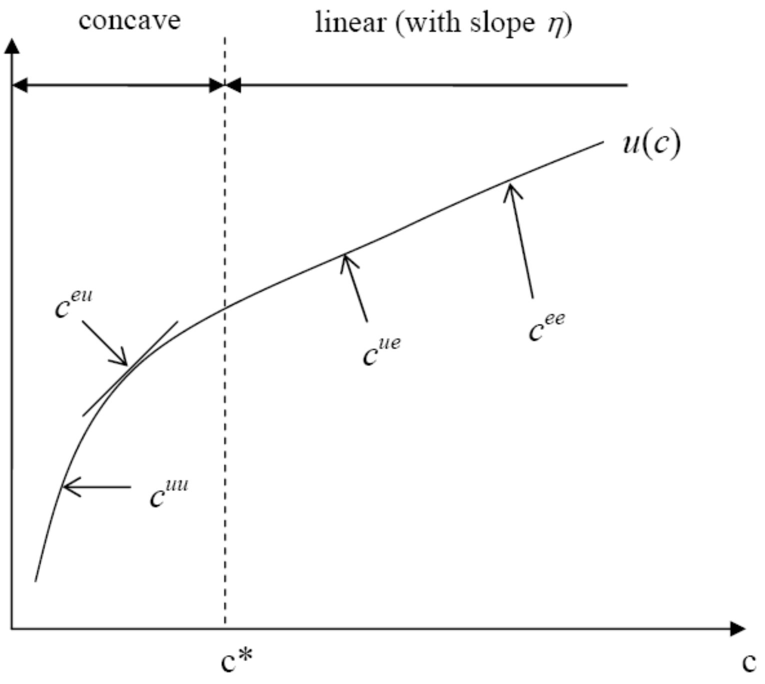

Figure 1: *Instant Utility Function of Impatient Households,* $u^I(c)$.

in (3) is binding for all unemployed, impatient households:

$$\textbf{Condition 2}: \forall i \in [0,\Omega],\ e_t^i = 0 \Rightarrow u'\left(c_t^i\right) > \mathbb{E}_t\left[\beta^I u'\left(c_{t+1}^i\right) R_{t+1}\right] \text{ and } a_t^i = -\mu. \tag{12}$$

Equations (11) and (12) have direct implications for the optimal asset holdings of employed households. By construction, a household that is employed at date $t$ has asset wealth $a_t^i R_{t+1}$ at the beginning of date $t+1$. If the household falls into unemployment at date $t+1$, then the borrowing constraint becomes binding and the household liquidates all assets. This implies that the household enjoys consumption:

$$c_{t+1}^i = \delta^I + \mu + a_t^i R_{t+1}, \tag{13}$$

and marginal utility $u^{I\prime}\left(\delta^I + \mu + a_{t+1}^i R_{t+1}\right)$.

There are now two cases to differentiate between, depending on whether or not this household faces a binding borrowing constraint at date $t$ (that is, when the household is still employed). If it does not, then $a_t^i > -\mu$ in (13), meaning that the household has formed a buffer of precautionary asset wealth in excess of the borrowing limit when still employed (with the buffer being of size $a_t^i + \mu > 0$). If it does, then $a_t^i = -\mu$ in (13) so that $c_{t+1}^i = \delta^I - \mu\left(R_{t+1} - 1\right)$, and the household will have consumed its entire (wage and asset) income at date $t$.

### 2.1.1 *The precautionary saving case*

If the borrowing constraint does not bind at date $t$, then $a_t^i > -\mu$ and the following Euler condition must hold at that date:

$$\eta = \beta^I \mathbb{E}_t \left[ \left[ (1 - s_{t+1}) \eta + s_{t+1} u^{I\prime} \left( \delta^I + \mu + a_t^i R_{t+1} \right) \right] R_{t+1} \right]. \tag{14}$$

The left hand side is the current marginal utility of this household, which is equal to $\eta$ under condition (11). The right hand side is expected, discounted future marginal utility, with marginal utility at date $t+1$ being broken into the two possible employment statuses that this household may experience at that date, weighted by their probabilities of occurrence. If the household stays employed at date $t+1$, which occurs with probability $1 - s_{t+1}$, it enjoys marginal utility $\eta$ (by (11)); if the household falls into unemployment, which occurs with probability $s_{t+1}$, assets are liquidated (by (12)) and the household enjoys marginal utility $u^{I\prime} \left( \delta^I + \mu + a_t^i R_{t+1} \right)$. Since (14) pins down $a_t^i$ as a function of *aggregate* variables only (i.e., $s_{t+1}$ and $R_{t+1}$), asset holdings are symmetric across employed households:

$$\forall i \in [0, \Omega], \; e_t^i = 1 \Rightarrow a_t^i = a_t. \tag{15}$$

To get further insight into how unemployment risk affects precautionary wealth, it is useful to substitute (15) into (14) and rewrite the Euler equation for employed households as:

$$\beta^I \mathbb{E}_t \left[ \left( 1 + s_{t+1} \frac{u^{I\prime} \left( \delta^I + \mu + a_t R_{t+1} \right) - \eta}{\eta} \right) R_{t+1} \right] = 1. \tag{16}$$

Consider, for the sake of the argument, the effect of a fully predictable increase in $s_{t+1}$ holding $R_{t+1}$ constant. The direct effect is to increase $1 + s_{t+1}[u^{I\prime} \left( \delta^I + \mu + a_t R_{t+1} \right) - \eta]/\eta$, since the proportional change in marginal utility associated with becoming unemployed, $[u^{I\prime} \left( \delta^I + \mu + a_t R_{t+1} \right) - \eta]/\eta$, is positive (see Figure 1). Hence, $u^{I\prime} \left( \delta^I + \mu + a_t R_{t+1} \right)$ must go down for (16) to hold, which is achieved by increasing date $t$ asset holdings, $a_t$.

### 2.1.2 *The hand-to-mouth case*

In the case where the borrowing constraint is binding for all impatient households, then by equation (2) the consumption levels of employed and unemployed households are, respectively, $w_t (1 - \tau_t) - \mu (R_t - 1)$ and $\delta^I - \mu (R_t - 1)$, meaning that all impatient households consume their income in every period. Our model thus contains the "hand-to-mouth" model as a special case. As discussed below, this corner scenario arises most notably when either direct unemployment insurance is sufficiently generous (so households do not self-insure) or impatient households' discount factor is sufficiently low (i.e., households are too impatient to save).

### 2.1.3 *Aggregation*

The analysis above implies that, under conditions (11) and (12), the cross-sectional distribution of wealth amongst impatient households at any point in time has at most two states. There are exactly two wealth states ($-\mu$ and $a_t > -\mu$) if the borrowing constraint is binding for unemployed households but not for employed households, and there is exactly one wealth state ($-\mu$) if the constraint is binding for all impatient households. This in turn implies that the economy is populated by at most *four* types of impatient households since, by equation (2), the type of household depends on both beginning-of-period and end-of-period asset wealth. We call these types "$ij$", where $i, j \in [e, u]$. Here $i$ ($j$) refers to the household's employment status in the previous (current) period. For example, a "$ue$ household" is currently employed but was unemployed in the previous period, and its consumption at date $t$ is $c_t^{ue}$. The individual consumption levels are:

$$c_t^{ee} = w_t^I (1 - \tau_t) + R_t a_{t-1} - a_t, \quad c_t^{eu} = \delta^I + \mu + R_t a_{t-1}, \tag{17}$$

$$c_t^{ue} = w_t^I (1 - \tau_t) - a_t - \mu R_t, \quad c_t^{uu} = \delta^I + \mu - \mu R_t, \tag{18}$$

where $a_t$ is given by (16) in the precautionary-saving case and by $-\mu$ in the hand-to-mouth case. Hence, in the latter case, $c_t^{ee} = c_t^{ue}$ and $c_t^{eu} = c_t^{uu}$. Finally, defining $\omega^{ij}$ to be the measure of impatient households of type $ij$ in the economy at date $t$, gives us the labor market flows:

$$\omega_t^{ee} = \Omega (1 - s_t) \left(\omega_{t-1}^{ee} + \omega_{t-1}^{ue}\right), \ \omega_t^{eu} = \Omega s_t \left(\omega_{t-1}^{ee} + \omega_{t-1}^{ue}\right), \tag{19}$$

$$\omega_t^{uu} = \Omega (1 - f_t) \left(\omega_{t-1}^{eu} + \omega_{t-1}^{uu}\right), \ \omega_t^{ue} = \Omega f_t \left(\omega_{t-1}^{eu} + \omega_{t-1}^{uu}\right). \tag{20}$$

The limited cross-sectional heterogeneity that prevails across impatient households implies that we can exactly aggregate their asset holding choices. By (12) and (15), the total asset holdings by impatient households is:

$$A_t^I \equiv \Omega \sum_{e=0,1} \int_{\tilde{a}=-\mu}^{+\infty} a_t (\tilde{a}, e) \, \mathrm{d}F_t (\tilde{a}, e) = \Omega \left[n_t a_t - (1 - n_t) \mu\right], \tag{21}$$

which can be substituted into market-clearing condition (8). Similarly, aggregating individual consumption levels (17) and (18) given the distribution of types in (19) and (20), we find total consumption by impatient households to be:

$$\begin{aligned} C_t^I &\equiv \Omega \sum_{e=0,1} \int_{\tilde{a}=-\mu}^{+\infty} c_t\left(\tilde{a}, e\right) \mathrm{d}F_t\left(\tilde{a}, e\right) \qquad (22) \\ &= \underbrace{\Omega\left[n_t w_t^I\left(1-\tau_t\right)+\left(1-n_t\right)\delta^I\right]+\left(R_t-1\right)A_{t-1}^I}_{\text{net income}} - \underbrace{\Omega\Delta\left[n_t\left(a_t+\mu\right)\right]}_{\text{change in asset wealth}}, \end{aligned}$$

where $A_{t-1}^I$ is given by (**??**) and $\Delta$ is the difference operator (so that $\Delta A_t^I = \Omega\Delta\left[n_t\left(a_t+\mu\right)\right]$).

Equation (21) summarizes the determinants of total consumption by impatient households in the economy. At date $t$, their aggregate net income is given by past asset accumulation and current factor payments–and hence taken as given by the households in the current period. The change in their total asset holdings, $\Omega\Delta\left[n_t\left(a_t+\mu\right)\right]$, depends on both the change in the number of precautionary savers, $\Omega n_t$ (the "extensive" asset holding margin), and the assets held by each of them, $a_t$ (the "intensive" margin). The former is determined by employment flows and is thus beyond the households' control, while the latter is their key choice variable. In the *precautionary saving* case, $a_t$ is given by (16) and hence increases when labor market conditions are expected to worsen (i.e., $s_{t+1}$ is expected to fall), which contributes to a decrease in $C_t^I$. In the *hand-to-mouth* (*HTM*) case, we simply have $a_t = -\mu$, so that $A_t^{I,HTM} = -\mu\Omega$ and

$$C_t^{I,HTM} = \Omega\left[n_t w_t^I\left(1-\tau_t\right)+\left(1-n_t\right)\delta^I-\mu\left(R_t-1\right)\right], \qquad (23)$$

implying that only current labor market conditions affect $C_t^{I,HTM}$ via their effect on $n_t$. Comparing (22) and (23), we get:

$$C_t^I = C_t^{I,HTM} + \Omega\left[R_t n_{t-1}\left(a_{t-1}+\mu\right) - n_t\left(a_t+\mu\right)\right].$$

This expression shows how total consumption by impatient households differs across the hand-to-mouth and the precautionary-saving cases. In the first case, only current labor market conditions $n_t$ (in addition to the factor payments $w_t^I\left(1-\tau_t\right)$, $R_t$) affect $C_t^I$. In the second case, the same effects are at work but *future* labor market conditions also matter because they affect $a_t$. This suggests that the precautionary savings model may display *more* consumption volatility than the hand-to-mouth model, provided that labor market conditions are sufficiently persistent. This will be confirmed in the quantitative analysis of Section 3.

## 2.2 *Existence Conditions and Steady State*

### 2.2.1 *Existence conditions*

The equilibrium with limited cross-sectional heterogeneity described so far exists provided that two conditions are satisfied. First, the postulated ranking of consumption levels for impatient households in (11) must hold in equilibrium. Second, unemployed, impatient households must face a binding borrowing constraint (see (12)). From equations (17)–(18), and the fact that $a_t \geq -\mu$ (with equality in the hand-to-mouth case), we have $c_t^{uu} \leq c_t^{eu}$ and $c_t^{ee} \geq c_t^{ue}$. Hence, a necessary and sufficient condition for (11) to hold is $c_t^{eu} < c^* < c_t^{ue}$, that is:

$$\delta^I + \mu + a_{t-1}R_t < c^* < w_t^I (1 - \tau_t) - a_t - \mu R_t. \tag{24}$$

Unemployed, impatient households can be of two types, $uu$ and $eu$, and we need both to face a binding borrowing constraint in equilibrium. However, since $c_t^{uu} \leq c_t^{eu}$ (and hence $u^{I\prime}(c_t^{uu}) \geq u^{I\prime}(c_t^{eu})$), a necessary and sufficient condition for both types to be constrained is:

$$u^{I\prime}(c_t^{eu}) > \beta^I \mathbb{E}_t \left[ \left[ f_{t+1} u^{I\prime}(c_t^{ue}) + (1 - f_{t+1}) u^{I\prime}\left(c_{t+1}^{uu}\right) \right] R_{t+1} \right], \tag{25}$$

where the right hand side of the inequality is the expected, discounted marginal utility of an $eu$ household that is contemplating the possibility of either remaining unemployed (with probability $1 - f_{t+1}$) or finding a job (with probability $f_{t+1}$). Under the conjectured equilibrium we have $u^{I\prime}(c_t^{ue}) = \eta$ and $c_{t+1}^{uu} = \delta^I + \mu(1 - R_{t+1})$, so (25) becomes:

$$u^{I\prime}\left(\delta^I + \mu + a_{t-1}R_t\right) > \beta^I \mathbb{E}_t \left[ \left\{ f_{t+1}\eta + (1 - f_{t+1}) u^{I\prime}\left[\delta^I + \mu(1 - R_{t+1})\right] \right\} R_{t+1} \right]. \tag{26}$$

In what follows, we compute the steady state of our conjectured equilibrium and derive a set of necessary and sufficient conditions for (24) and (26) to hold in the absence of aggregate shocks. By continuity, they will also hold in the stochastic equilibrium, provided that the magnitude of aggregate shocks is not too large. In what follows we exclusively focus on the case of "small" aggregate shocks in the sense that all macro variables are assumed to remain in the vicinity of their steady state values. The case of "large" shocks, and the conditions under which they are consistent with limited cross-sectional heterogeneity, is discussed and analyzed formally in the separate technical appendix (Section 2). In some cases, the full nonlinear dynamics of the model admits a two-state Markovian representation, making it straightforward to run stochastic simulations of the model and to check its existence conditions. When the baseline model does not literally admit such a representation (this occurs, for example, whenever $\Omega < 1$), then a open-economy version of the same model does and can be solved and simulated in a similar way. In both cases, we find the support of admissible exogenous aggregate shock processes such that (24)–(26) hold to be large–much larger than the typical business cycle shock (see the technical appendix for details).

### 2.2.2 *Steady state*

In the steady state, the real interest rate is determined by the discount rate of the most patient households, so that $R^* = 1/\beta^P$ (see (6)). From equations (1) and (7), the steady state levels of employment and capital per effective labor unit are**:**

$$n^* = f^*/\left(f^* + s^*\right),\ k^* = g'^{-1}\left(1/\beta^P - 1 + \nu\right). \tag{27}$$

A key variable in the model is the level of asset holdings that employed, impatient households hold as a buffer against unemployment risk. If the borrowing constraint is binding in the steady state, then they never hold any wealth. The interior solution to the steady state counterpart of (16) (where $R^* = 1/\beta^P$) gives the individual asset holdings:

$$\tilde{a}^* = \beta^P \left[\left(u^{I\prime -1}\left\{\eta\left[1 + \left(\beta^P - \beta^I\right)/\beta^I s^*\right]\right\}\right) - \delta^I - \mu\right]. \tag{28}$$

The borrowing constraint is binding whenever $\tilde{a}^* < -\mu$. Hence, the actual steady-state wealth level of employed, impatient households is given by:

$$a^* = \max\left[-\mu, \tilde{a}^*\right], \tag{29}$$

which encompasses both the precautionary-saving and hand-to-mouth cases discussed above. Finally, (8) and (21) imply that steady state (total) asset holdings by impatient and patient households are $A^{I*} = \Omega\left[n^* a^* - (1 - n^*)\,\mu\right]$ and $A^{P*} = K^* - A^{I*} = \left[\Omega + (1 - \Omega)\,\kappa\right] n^* k^* - A^{I*}$, respectively. It then follows from (27) that the cross-sectional wealth distribution is summarized by the following wealth shares:

$$\frac{A^{I*}}{K^*} = \frac{\Omega\left(a^* - \mu s^*/f^*\right)}{g'^{-1}\left(1/\beta^P - 1 + \nu\right)}, \qquad \frac{A^{P*}}{K^*} = 1 - \frac{A^{I*}}{K^*}. \tag{30}$$

Equations (28) and (29) are informative about the conditions under which households find it worthwhile to hold a buffer stock of wealth in excess of the borrowing limit. They do so whenever:

$$s^* \left[\frac{u'\left(\delta^I + \mu - \mu/\beta^P\right) - \eta}{\eta}\right] > \frac{\beta^P - \beta^I}{\beta^I}. \tag{31}$$

The greater the relative impatience of impatient households, as measured by $\left(\beta^P - \beta^I\right)/\beta^I$, the less likely inequality (31) will hold. The greater the subjective cost of an unbuffered

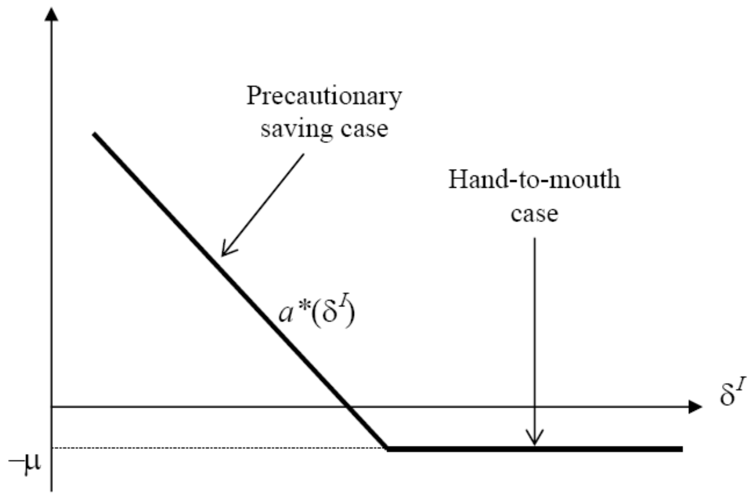

Figure 2: *Unemployment Insurance and Precautionary Saving.*

transition from employment (where marginal utility is $\eta$) to unemployment (where marginal utility, without buffer-stock saving, is $u'\left(\delta^I + \mu - \mu/\beta^P\right)$), weighted by the probability of this transition occurring ($s^*$), the more likely it will hold. In particular, the greater the unemployment benefit $\delta^I$, the lower the subjective cost of falling into unemployment and the weaker the incentive to hold a buffer stock. Formally, $a^*\left(\delta^I\right)$ is a nonincreasing, continuous piecewise linear function with a kink at the value of $\delta^I$ for which $\tilde{a}^* = -\mu$ (see Figure 2).

The following proposition establishes the conditions on the deep parameters of the model under which a steady state with limited cross-sectional heterogeneity exists. Provided that the aggregate shocks have a sufficiently small magnitude, the same conditions will ensure the existence of a stochastic equilibrium with similarly limited heterogeneity.

**Proposition 1**. *Assume that i) there are no aggregate shocks*; *ii) unemployment insurance is incomplete (i.e.,* $\delta^I < w^{I*}(1-\tau^*)$*); and iii) the following inequality holds:*

$$\eta\left(1+\frac{\beta^P-\beta^I}{\beta^I s^*}\right) >$$
$$\max\left\{\frac{\beta^I}{\beta^P}\left[f^*\eta + (1-f^*)\,u'\left(\delta^I - \mu\left(\frac{1}{\beta^P}-1\right)\right)\right], u'\left[\frac{w^{I*}(1-\tau^*)+\beta^P\delta^I}{1+\beta^P} - \mu\left(\frac{1}{\beta^P}-1\right)\right]\right\},$$

*where*

$$\tau^* = \frac{\left[\Omega\delta^I + (1-\Omega)\,\delta^P\right](1-n^*)}{\left[\Omega + (1-\Omega)\,\kappa\right] n^* w^{I*}}, \tag{32}$$

$w^{I*} = g(k^*) - k^* g'(k^*)$, *and* $(n^*, k^*)$ *are given by (27). Then, it is always possible to find a utility threshold* $c^*$ *such that the conjectured limited-heterogeneity equilibrium described above exists. In this equilibrium,* $a^* = -\mu$ $(a^* > -\mu)$ *if (31) holds (does not hold).*

**Proof**. First, the steady state counterpart of (26) is:

$$a^* < \beta^P u^{I\prime -1}\left(\frac{\beta^I}{\beta^P}\left\{f\eta + (1-f)\, u'\left[\delta^I + \mu\left(\frac{\beta^P - 1}{\beta^P}\right)\right]\right\}\right) - \beta^P\left(\delta^I + \mu\right). \qquad (33)$$

Second, the steady state counterpart of (24) is $\delta^I + \mu + a^*/\beta^P < c^* < w^{I*}(1-\tau_t) - a^* - \mu/\beta^P$. A sufficient condition for the existence of a threshold $c^*$ is thus $\delta^I + \mu + a^*/\beta^P < w^{I*}(1-\tau_t) - a^* - \mu/\beta^P$, or:

$$a^* < \beta^P \Gamma / \left(1 + \beta^P\right) - \mu, \qquad (34)$$

where $\Gamma \equiv w^{I*}(1-\tau^*) - \delta^I = (1-\tau^*)\left[g(k^*) - k^* g'(k^*)\right] - \delta^I$ is a strictly positive constant that only depends on the deep parameters of the model (see (27) and (32)). Inequalities (33) and (34) hold for $a^* = -\mu$ (the hand-to-mouth case). Otherwise, $a^*$ is given by (28) (the precautionary saving case). Substituting this value of $a^*$ into (33) and (34) and rearranging gives the inequality in the proposition.■

The inequalities in Proposition 1 ensures that two properties hold at the steady state. First, the candidate equilibrium features at most two possible wealth levels for impatient households ($-\mu$ for the unemployed and $a^* \geq -\mu$ for the employed). Second, the implied ranking of individual consumption levels is such that we can "reverse-engineer" an instant utility function for these households of the form depicted in Figure 1. These inequalities are straightforward to check once specific values are assigned to the deep parameters of the model. As we argue in Section 3, the inequalities are satisfied for plausible values when we calibrate the model to the US economy. This is because our limited-heterogeneity equilibrium requires that impatient unemployed households be borrowing-constrained (i.e., they would like to borrow against future income but are prevented from doing so), and that impatient employed households accumulate too little wealth in equilibrium–so little that their wealth will be exhausted after the first quarter of unemployment. In the US, the quarter-to-quarter probability of leaving unemployment is high and the replacement ratio is relatively low. This causes the expected income of the unemployed to be larger than current income, thereby making these households willing to borrow. On the other hand, the US distribution of wealth is fairly unequal, leading a large fraction of the population (the impatient in our model) to hold a very small fraction of total wealth.

### 2.2.3 *An approximate asset holding rule*

We conclude this section by stressing that when (employed) impatient households do form precautionary saving, then local time-variations in the job-loss probability $s_{t+1}$ have a *first-*

*order* effect on precautionary asset accumulation at the individual level, $a_t$. This is because, even without aggregate risk, a change in employment status from employed to unemployed at date $t+1$ is associated with a discontinuous drop in individual consumption and, hence, with an infra-marginal rise in marginal utility from $\eta$ to $u^{I\prime}(c^{eu}) > \eta$. The probability $s_{t+1}$ weights this possibility in the employed households' Euler equation (see (16)), so even small changes in $s_{t+1}$ have a sizable impact on asset holdings and consumption choices. To illustrate this point, we let hats denote level-deviations from the steady state (i.e., $\hat{x}_t = x_t - x^*$), and use (16) to arrive at the following approximation of the optimal asset-holding rule:[6]

$$\hat{a}_t \simeq \Gamma + \underbrace{\Gamma_s \mathbb{E}_t\left(\hat{s}_{t+1}\right) + \Gamma_R \mathbb{E}_t(\hat{R}_{t+1})}_{\text{first-order terms}} + \underbrace{\frac{\Gamma_{s^2}}{2}\mathbb{E}_t\left(\hat{s}_{t+1}^2\right) + \frac{\Gamma_{R^2}}{2}\mathbb{E}_t(\hat{R}_{t+1}^2) + \frac{\Gamma_{sR}}{2}\mathbb{E}_t\left(\hat{s}_{t+1}\hat{R}_{t+1}\right)}_{\text{second-order terms}}, \quad (35)$$

where the Γs are constants. The first-order responsiveness of precautionary wealth ($\hat{a}_t = a_t - a^*$) to unemployment risk (as measured by $\hat{s}_{t+1} = s_{t+1} - s^*$) is given by the composite parameter:

$$\Gamma_s = \frac{\left(\beta^P - \beta^I\right)\left[\beta^P\left(\delta^I + \mu\right) + a^*\right]}{\left[\beta^P - \beta^I\left(1 - s^*\right)\right] s^* \sigma^I\left(c^{eu*}\right)} > 0,$$

where $a^*$ is given by (28), $c^{eu*} = \delta^I + \mu + a^* R^*$ is the steady state counterpart of $c_t^{eu}$ in (17), and $\sigma^I\left(c^{eu*}\right) \equiv -c^{eu*} u^{I\prime\prime}\left(c^{eu*}\right)/u^{I\prime}\left(c^{eu*}\right)$. The greater $\Gamma_s$, the stronger the response of individual asset holdings to shocks affecting the job-loss rate; such shocks thus affect total consumption (by equation (22)) even when the second-order terms of the model's aggregate dynamics are neglected. Note that this first-order effect of time-varying idiosyncratic risk does not depend on the fact that the constraint be immediately binding when a worker falls into unemployment. What matters is that the job loss be associated with a infra-marginal drop in individual consumption, which also occurs when the worker does not immediately face a binding constraint, but may face it in the future (and hence spreads the consumption fall over several periods). We analyze this possibility in the separate technical appendix to the paper (Section 1). The property that time-varying idiosyncratic risk affects savings at the first order distinguishes models with borrowing limits–included ours–from those that root the precautionary motive into households' "prudence" (i.e., positive third-order derivative, see Kimball, 1990) and wherein time-variations in precautionary savings follow from changes in the second-order term of future marginal utility (see, e.g., Gourinchas and Parker, 2001; Parker and Preston, 2005).[7]

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[6] See the technical appendix for details, including the expressions for all coefficients in the rule.

[7] It is apparent from (16) that a mean-preserving increase in employed households' uncertainty about future labor income, taking the form of an increase in $s_{t+1}$ (and a corresponding rise in $w_{t+1}$ to keep expected income constant), increases asset holdings. This is the usual definition of "precautionary saving."

# 3 Time-Varying Precautionary Saving and Consumption Fluctuations

The model above implies that some households respond to countercyclical changes in unemployment risk by raising precautionary wealth and thus by decreasing consumption more than they would have without the precautionary motive. We now assess the extent of this effect on total consumption when realistic unemployment shocks are fed into our model economy. To do so, we compute the response of aggregate consumption and output to aggregate shocks implied by our baseline model. We then compare it with the data and several alternative benchmarks (namely, the hand-to-mouth model, the representative-agent model, and the Krusell and Smith (1998) model).

## 3.1 *Summary of the Baseline Precautionary-Saving Model*

The precautionary-saving model includes three forcing variables $(z_t, f_t, s_t)$ and ten endogenous variables $(n_t, k_t, C_t^I, C_t^P, A_t^P, A_t^I, a_t, R_t, w_t^I, \tau_t)$, linked through the following equations:

$$\beta^I \mathbb{E}_t \left[ \left[ 1 + s_{t+1} \left( u^{I\prime} \left( \delta^I + \mu + a_t R_{t+1} \right) - \eta \right) / \eta \right] R_{t+1} \right] = 1, \tag{EE-I}$$

$$C_t^I + A_t^I = \Omega \left[ n_t w_t^I \left( 1 - \tau_t \right) + \left( 1 - n_t \right) \delta^I \right] + R_t A_{t-1}^I, \tag{BC-I}$$

$$A_t^I = \Omega \left[ n_t a_t - \left( 1 - n_t \right) \mu \right], \tag{A-I}$$

$$\beta^P \mathbb{E}_t \left( u^{P\prime} \left[ \left( C_{t+1}^P / \left( 1 - \Omega \right) \right) R_{t+1} / u^{P\prime} \left( C_t^P / \left( 1 - \Omega \right) \right) \right] \right) = 1, \tag{EE-P}$$

$$C_t^P + A_t^P = \left( 1 - \Omega \right) \left[ \kappa n_t w_t^I \left( 1 - \tau_t \right) + \left( 1 - n_t \right) \delta^P \right] + R_t A_{t-1}^P, \tag{BC-P}$$

$$R_t = z_t g' \left( k_t \right) + 1 - \nu, \tag{IR}$$

$$w_t^I = z_t \left[ g \left( k_t \right) - k_t g' \left( k_t \right) \right], \tag{WA}$$

$$A_{t-1}^P + A_{t-1}^I = \left[ \Omega + \left( 1 - \Omega \right) \kappa \right] n_t k_t, \tag{CM}$$

$$\tau_t n_t w_t^I \left[ \Omega + \left( 1 - \Omega \right) \kappa \right] = \left( 1 - n_t \right) \left[ \Omega \delta^I + \left( 1 - \Omega \right) \delta^P \right], \tag{UI}$$

$$n_t = \left( 1 - n_{t-1} \right) f_t + \left( 1 - s_t \right) n_{t-1}. \tag{EM}$$

Equations (EE-I)–(A-I) are the Euler condition and aggregate budget constraint for impatient households–as described in Section 2.1. Equations (EE-P) and (BC-P) are the same conditions for patient households, as described in Section 1.1, where $w_t^P$ has been replaced by its equilibrium value, $\kappa w_t^I$. (IR) follows from (7), with the factor price frontier under CRS giving $w_t^I$ in (WA). (CM) is the market-clearing condition for capital, which follows from substituting (21) into (8). Finally, (UI) is the balanced-budget condition for the unemployment insurance scheme (where again $w_t^P = \kappa w_t^I$ has been substituted into (10)), and (EM) is the law of motion for employment. The model above can be linearized to give:

$$\hat{a}_t = \Gamma_s \mathbb{E}_t\left(\hat{s}_{t+1}\right) + \Gamma_R \mathbb{E}_t(\hat{R}_{t+1}), \tag{EE-I*}$$

$$\hat{C}_t^I + \hat{A}_t^I = \Omega\left\{n^*\left(1-\tau^*\right)\hat{w}_t^I + \left[w^{I*}\left(1-\tau^*\right) - \delta^I\right]\hat{n}_t - n^* w^{I*}\hat{\tau}_t\right\} + A^{I*}\hat{R}_t + R^*\hat{A}_{t-1}^I, \tag{BC-I*}$$

$$\hat{A}_t^I = \Omega n^* \hat{a}_t + \Omega\left(a^* + \mu\right)\hat{n}_t, \tag{A-I*}$$

$$\mathbb{E}_t[\Delta\hat{C}_{t+1}^P / C^{P*}] = \left(\sigma^P / R^*\right)\mathbb{E}_t[\hat{R}_{t+1}], \tag{EE-P*}$$

$$\begin{aligned}\hat{C}_t^P + \hat{A}_t^P = {} & (1-\Omega)\left\{n^*\left(1-\tau^*\right)\kappa\hat{w}_t^I + \left[\kappa w^{I*}\left(1-\tau^*\right) - \delta^P\right]\hat{n}_t - n^*\kappa w^{I*}\hat{\tau}_t\right\} \\ & + A^{P*}\hat{R}_t + R^*\hat{A}_{t-1}^P\end{aligned} \tag{BC-P*}$$

$$\hat{R}_t = \left[g'\left(k^*\right)\right]\hat{z}_t + g''\left(k^*\right)\hat{k}_t, \tag{IR*}$$

$$\hat{w}_t^I = \left[g\left(k^*\right) - k^* g'\left(k^*\right)\right]\hat{z}_t - k^* g''\left(k^*\right)\hat{k}_t, \tag{WA*}$$

$$\hat{A}_{t-1}^P + \hat{A}_{t-1}^I = \left[\Omega + (1-\Omega)\kappa\right]\left(n^*\hat{k}_t + k^*\hat{n}_t\right), \tag{CM*}$$

$$\tau^* n^* \hat{w}_t^I + \tau^* w^{I*}\hat{n}_t + n^* w^{I*}\hat{\tau}_t = -\left[\frac{\Omega\delta^I + (1-\Omega)\delta^P}{\Omega + (1-\Omega)\kappa}\right]\hat{n}_t, \tag{UI*}$$

$$\hat{n}_t = \left(1-n^*\right)\hat{f}_t - f^*\hat{n}_{t-1} + \left(1-s^*\right)\hat{n}_{t-1} - n^*\hat{s}_t, \tag{EM*}$$

where hats denote level-deviations from the steady state and $(\Gamma_s, \Gamma_R)$ are as in equation (35).

## 3.2 *Alternative Benchmarks*

### 3.2.1 *Hand-to-mouth model*

When condition (31) holds, all impatient households face a binding borrowing constraint in every period, so that $a_t = -\mu$ and $A_t^I = -\mu\Omega$ for all $t$ (see Section 2.1 above). The resulting dynamics are obtained by removing equation (EE-I) from the baseline model and by imposing $a_t = -\mu$ in equation (A-I). Moreover, since $\hat{A}_{t-1}^I = 0$ the linearized hand-to-mouth model is composed of equations (EE-P*), (BC-P*), (IR*), (WA*), (UI*) and (EM*), together with following modifications of (BC-I*) and (CM*):

$$\begin{aligned}\hat{C}_t^I &= \Omega n^*\left(1-\tau^*\right)\hat{w}_t^I + \Omega\left[w^{I*}\left(1-\tau^*\right) - \delta^I\right]\hat{n}_t - \Omega n^* w^{I*}\hat{\tau}_t + -\mu\Omega\hat{R}_t, \\ \hat{A}_{t-1}^P &= \left(\Omega + (1-\Omega)\kappa\right)\left(n^*\hat{k}_t + k^*\hat{n}_t\right).\end{aligned}$$

### 3.2.2 *Representative-agent model*

The comparable representative-agent model is obtained by setting $\Omega^{RA} = 0$ (so that all households are identical and fully insured), and $\kappa^{RA} = \Omega + (1-\Omega)\kappa$ (so that average labor productivity is the same as in the baseline model). With $\beta^P$ unchanged, $R^*$, $k^*$, as well as steady-state total wealth $(\Omega + (1-\Omega)\kappa) n^* k^*$ (see (8)), remain unchanged. The model is composed of equations (EE-P*), (IR*), (WA*), (EM*), and:

$$\hat{C}_t^P + \hat{A}_t^P = n^*(1-\tau^*)\kappa \hat{w}_t^I + \left[\kappa w^{I*}(1-\tau^*) - \delta^P\right]\hat{n}_t, -n^*\kappa w^{I*}\hat{\tau}_t + A^{P*}\hat{R}_t + R^*\hat{A}_{t-1}^P,$$

$$\hat{A}_{t-1}^P = \kappa^{RA}(n^*\hat{k}_t + k^*\hat{n}_t),$$

$$\tau^* n^* \hat{w}_t^I + \tau^* w^{I*}\hat{n}_t + n^* w^{I*}\hat{\tau}_t = -\left(\delta^P/\kappa^{RA}\right)\hat{n}_t.$$

### 3.2.3 *Krusell-Smith model*

We also compare the quantitative properties of our model with the stochastic-beta version of the Krusell and Smith (1998) heterogeneous-agent model. We focus on the stochastic-beta model for essentially two reasons: first, because it incorporates discount factor heterogeneity, which, as with our model, potentially generates a substantial amount of wealth dispersion; and second, because it is the model variant that quantitatively differs most from the full-insurance model. In order to meaningfully compare the stochastic properties of our baseline model with the Krusell-Smith model, we rescale the size of the aggregate shocks (TFP and unemployment) in the latter so that it produces the same output volatility as our baseline model (see Section 3.4.1 for details).

## 3.3 *Calibration*

### 3.3.1 *Idiosyncratic risk and insurance*

The period is a quarter. We set the steady state values of $f^*$ and $s^*$ to their quarter-to-quarter, post-war averages (see our technical appendix for a description of all the series used in this section). In a narrow sense, the gross replacement ratio $\delta^j/w^{*j}$, $j = I, P$, is the income provided by the unemployment insurance scheme and should thus be set between 0.4 and 0.5 for the US (see Shimer, 2005 or Chetty, 2008). However, households also benefit from other sources of insurance (family, friends, etc.) so we take this into account by calibrating $\delta^j/w^{*j}$ so as to generate a plausible level of consumption insurance for the period following the job loss (i.e., the following quarter here). Cochrane (1991) argues that the average consumption growth of consumers experiencing an involuntary job loss is 25 percentage points lower than those who do not. Gruber (1997) focuses on the impact of UI benefits on the size of the consumption fall of households having experienced a job loss. He finds an average fall of about 7 percent.[8] We set the baseline value of $\delta^j/w^{*j}$ to 0.6 (rather than, say, 0.5) which, together with the other parameters of the model, produces a consumption growth differential of 14.26% for the average household.[9] As we show below, it turns out that the calibrated value of $\delta^j/w^{*j}$ mainly affects the cross-sectional distribution of wealth but has a limited effect on aggregate volatility statistics. In our baseline scenario we assume that impatient households cannot borrow (i.e., $\mu = 0$) and we then relax this constraint in our sensitivity analysis.

### 3.3.2 *Preferences and technology*

Impatient households in our baseline precautionary-saving model are somewhat wealthier than pure hand-to-mouth consumers. Since we are moving one step up in the wealth distribution–relative to households facing a binding debt limit and holding not wealth–,we calibrate their share at a level that is no less than the available estimates of the share of hand-to-mouth households in the US economy. Estimates range from 15% to 60% (see Campbell and Mankiw, 1989; Iacoviello, 2005; Gali et al., 2007; Mertens and Ravn, 2011; and Kaplan and Violante, 2012); we thus set $\Omega = 0.6$ in our baseline specification. Since only a fraction $1 - n^*$ of such households face a binding borrowing constraint in the baseline, and given the calibrated steady-state transition rates $f^*$ and $s^*$, this implies a steady-state share of effectively borrowing-constrained households of $\Omega\left(1 - n^*\right) = 3.4\%$.[10] The discount factor of patient households, $\beta^P$ $(= 1/R^*)$, is set to 0.99, and their instant utility to $u^P\left(c\right) = \ln c$. The utility function of impatient households is:

$$u^I\left(c\right) = \begin{cases} \ln c & \text{for } c \leq 1.6 \\ \ln 1.6 + 0.504\left(c - 1.6\right) & \text{for } c > 1.6, \end{cases} \tag{36}$$

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[8] This strand of the microeconometric literature uses the PSID. As a consequence, it measures variations in *food* consumption (rather than total nondurable and services consumption) at the *yearly* (rather than quarterly) frequency. As a result, when a household is reported unemployed it may have been so for more than a quarter.

[9] Patient households are fully insured and hence experience no fall in consumption when becoming unemployed. Hence, the average proportional consumption drop associated with this event is $\Omega\left(c^{e*} - c^{eu*}\right)/c^{e*}$, where $c^{e*} \equiv f^*\left(1 - n^*\right)c^{ue*} + (1 - s)n^*c^{ee*}$ is the average consumption of employed, impatient households. Given that our calibrated replacement ratio is perfectly symmetric across households, we are implicitly ignoring the potential redistributive effects of the unemployment insurance scheme.

[10] Of course, in our hand-to-mouth benchmark the share of effectively constrained households increases from $\left(1 - n^*\right)\Omega$ $(= 3.4\%)$ to $\Omega$ $(= 60\%)$ since all impatient households, including the employed, are then constrained.

which satisfies the assumptions in Section 2.1 (with $\eta = 0.504$ and $c^* = 1.6$). The chosen value of $\eta$ is equal to the steady state marginal utility of $ee$ households (by far the most numerous amongst the impatient) if they had the same instant utility function as patient households, given the other parameters.[11] This choice was meant to minimize differences in asset holding behavior purely due to differences in instant utility functions. Note that $u^I(c)$ is continuous and (weakly) concave but not differentiable over the entirety of $[0, \infty)$ since $u^{I\prime}(1.6) > 0.504$. However, it can be made so by equating the right and left derivatives of $u^I(\cdot)$ in an arbitrarily small neighborhood of $c^*$ while preserving concavity. We set $\beta^I$ to match the wealth share of the $\Omega\%$ poorest households, given the other parameters. We focus on liquid wealth, since our analysis pertains to the part of households' net worth that can readily be used for current (nondurable) consumption. A value of $\beta^I = 0.972$ produces a wealth share of 0.30% for the poorest 60% of households, matching the corresponding quantile of the distribution of liquid wealth in the Survey of Consumer Finances (see our technical appendix for details).

The production function is $Y_t = z_t K_t^\alpha \left(n_t^I + \kappa n_t^P\right)^{1-\alpha}$, with $\alpha = 1/3$, and the depreciation rate is $\nu = 2.5\%$. The skill premium parameter $\kappa$ is set to 1.731. Given the other parameters, this value for the skill premium will produce a consumption share $C^{I*}/(C^{I*} + C^{P*})$ of 40.62% for the poorest 60% of households. This matches the cross-sectional distribution of nondurables in the Consumer Expenditure Survey and is also well in line with direct measures of the skill premium (Heathcote et al., 2010; Acemoglu and Autor, 2011).

Our baseline parameterization is summarized in Table 1. These parameters satisfy the existence conditions stated in Proposition . In particular, households that become unemployed exhaust their buffer stock of wealth within a quarter. Note that given the baseline values of $\Omega$ and $\kappa$, the representative-agent economy (in which $\Omega^{RA} = 0$) must be parameterized with a skill premium parameter $\kappa^{RA} = \Omega + (1 - \Omega)\kappa = 1.292$.

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[11] That is, $\eta$ solves $\eta = u^{P\prime}(c^{ee*}) = \left(w^{I*} + a^*\left(1 - 1/\beta^P\right)\right)^{-1}$.

| *Parameters* | Sym. | Value | *Steady state* (%) | Value | Data | Source |
|---|---|---|---|---|---|---|
| Share of impatient hous. | $\Omega$ | 0.6 | Unemployment rate | 5.54 | 5.54 | CPS |
| Disc. factor (patient) | $\beta^P$ | 0.990 | Liquid wealth share of | | | |
| Disc. factor (impatient) | $\beta^I$ | 0.972 | bottom $\Omega$% | 0.30 | 0.30 | SCF |
| Risk aversion | $\sigma^{I,P}$ | 1 | Consumption share of | | | |
| Replacement ratio | $\delta/w$ | 0.6 | bottom $\Omega$% | 40.62 | 40.62 | CEX |
| Borrowing limit | $\mu$ | 0.0 | Mean cons. fall after | | | |
| Skill premium parameter | $\kappa$ | 1.731 | unemployment shock | 14.23 | $[7, 25]$ | see text |
| Capital share | $\alpha$ | 1/3 | | | | |
| Depreciation rate | $\nu$ | 0.025 | | | | |
| Job-finding rate | $f^*$ | 0.8021 | | | | |
| Job separation rate | $s^*$ | 0.047 | | | | |

*Table 1. Baseline Model: Parameters and Implied Steady State.*

Note: The model matches the mean unemployment rate by construction. $\beta^I$, $\kappa$ are set so that the wealth and consumption shares of the model (column 5) match their empirical counterparts (column 6), given the other parameters of the model (see text for details).

## 3.4 *Aggregate Consumption Volatility*

### 3.4.1 *Experiment*

To compute the second-order moment properties of the various model specifications under consideration, we proceed as follows. We first estimate the joint behavior of the exogenous state vector over the entire postwar period using a VAR $\mathbf{x}_t = \sum_{j=1}^{4} \mathbf{A}_j \mathbf{x}_{t-j} + \varepsilon_t$. Here $\mathbf{x}_t = [\tilde{z}_t, \tilde{f}_t, \tilde{s}_t]'$ includes log-total factor productivity ("TFP" henceforth), made stationary using the HP-filter, as well as the job-finding and job-separation rates–also HP-filtered to remove their low-frequency movements. $\varepsilon_t$ is the $1 \times 3$ vector of residuals. This gives us $\mathbf{A}_j$, $j = 1...4$ as well as the covariance matrix $\mathbf{\Sigma} \equiv \text{Var}(\varepsilon_t)$. We then log-linearize the three model variants and solve for their state-space representation $\mathbf{z}_t = \mathbf{B}\mathbf{z}_{t-1} + \mathbf{x}_t$, where $\mathbf{z}_t$ is the relevant vector of endogenous variables. Last, we run stochastic simulations of each model with repeated shocks on $\mathbf{x}_t$ that have the same stochastic properties as the estimated VAR. Our results are almost identical when we consider a second-order rather than a first-order approximation of each model under consideration (the results are available upon request); this indicates that nonlinearities are not strong (at least under our baseline calibration) and confirms the importance of the first-order effects of time-varying idiosyncratic risk on individual savings.

As discussed above, we also compare these moments to those implied by a rescaled version of the stochastic-beta, Krusell-Smith model. More specifically, we simulate exactly the same model except that we specify the following support for the (two-state) aggregate exogenous state: unemployment varies from 5.4% (in the good state) to 8.6% (in the bad state) and TFP is equal to 1 in both states.[12] The transitions probabilities across aggregate states are as in the original paper, and the transitions across *individual* states (given those across aggregate states and the two values of the unemployment rate) are computed in the same way (see Krusell and Smith,1998, Section IV for details). We compute, for each relevant aggregate time series, the deviations from the sample mean resulting from the stochastic simulation of the model, and then report the corresponding statistics in Table 4 (Model 4). Our rescaling of the support of the exogenous state implies that the Krusell-Smith model now produces the same output volatility as our baseline precautionary-saving model–thereby making to two models comparable from a quantitative point of view.

### 3.4.2 *Results*

The quantitative merits of the four models under consideration–our baseline precautionary-saving model and the three alternative benchmarks introduced above–can be evaluated in light of their answers to the following two questions. First, does a particular model predict the correct amount aggregate (notably output) volatility, relative to the data? And second, at a given level of aggregate volatility, how consistent with the data are the *relative* volatilities of the components of output, i.e., consumption and investment? The second question is of particular interest because we expect the precautionary motive to substantially alter individual consumption-saving plans and the relative volatilities of consumption and investment, when compared to an economy without the precautionary motive.

It is apparent from the comparison of the data (row 1) and Model 2 in Table 2 that, under the shock process described above, the baseline precautionary-saving model tends to underestimate aggregate volatility (in output, consumption and investment). Such is also the case of the representative-agent and hand-to-mouth models. Importantly, given this overall underestimation of aggregate volatility our baseline model generates a substantial amount of consumption volatility (with a standard deviation of 0.79%, against 0.84% in the data). The reason for this was discussed in Section 2.1: in the precautionary-saving case, aggregate consumption responds not only to current labour market conditions (via their impact on

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[12] Plausible alternative rescaling schemes do not significantly affect aggregate volatilities.

current income), but also to future labour market conditions (via their impact on current precautionary wealth, see (22)–(23)). Provided that labour market conditions are persistent (which they are), the precautionary motive tends to reinforce the consumption response to aggregate shocks. In contrast, fluctuations in the precautionary motive are absent from both the hand-to-mouth and representative-agent models (Models 3 and 5), and as a result they generate relatively little consumption volatility (0.64% and 0.40%, respectively).

These results imply that our baseline precautionary-saving model departs significantly to the comparable representative-agent model as regards aggregate time-series behavior. Why is it so and, in particular, why does this result differ from that in the original analysis by Krusell and Smith (1998), who found their model to depart little from their comparable representative-agent model? As discussed above, our model is parameterized to match the share of *liquid* wealth held by the poorest 60%. In contrast, the original Krusell-Smith model is parameterized to fit the Lorenz curve for *net worth*. Since the latter is much less unequally distributed than liquid wealth, the Krusell-Smith model generates a much greater wealth share for the poorest 60% than does our baseline model. This implies that many "poor" households remain quite well self-insured in Krusell and Smith (1998) (and hence behave much like the permanent-income consumers), which is not the case of the workers in our model.

Looking at investment, one notices that what is gained in terms of consumption volatility is somewhat lost in terms of investment volatility, a dimension in which our baseline model fares particularly badly (but not as badly as the hand-to-mouth model). In particular, the representative-agent model generates a level of investment volatility that is both larger and closer to the data than that produced by our baseline model. This directly follows from the lack of buffer-stock saving behavior in the representative-agent economy. Indeed, in a recession aggregate savings fall more in the representative-agent economy, where households' savings decrease significantly, than in the in the precautionary-saving economy, where the rise in the job-loss rate strengthens the precautionary motive to save. Hence investment, and thereby the capital stock and aggregate output also fall more. Finally, comparing Model 2 and Model 4 reveals that the latter outperform ours in terms of volatility statistics (in the sense that consumption and investment volatility are both closer to the data). However, Model 4 clearly underestimate the consumption-output correlation.

| *Economies* | | *Statistics (%)* | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | share | standard deviations | | | | corr. with $Y$ | | | autocorrelations | | | |
| | | $\frac{A^{I*}}{K^*}$ | $Y$ | $C$ | $I$ | $\frac{\alpha Y}{K}$ | $C$ | $I$ | $\frac{\alpha Y}{K}$ | $Y$ | $C$ | $I$ | $\frac{\alpha Y}{K}$ |
| 1 | Data | 0.30 | 1.65 | 0.84 | 7.22 | 1.60 | 78 | 87 | 97 | 84 | 83 | 79 | 85 |
| 2 | Prec. saving | 0.30 | 1.29 | 0.79 | 3.68 | 1.32 | 82 | 92 | 93 | 81 | 57 | 73 | 80 |
| Alternative models | | | | | | | | | | | | | |
| 3 | Hand-to-mouth | 0.00 | 1.31 | 0.64 | 3.57 | 1.29 | 96 | 99 | 92 | 82 | 86 | 80 | 80 |
| 4 | Krusell-Smith | 5.3 | 1.29 | 0.84 | 5.15 | 1.10 | 16 | 89 | 57 | 83 | 90 | 72 | 74 |
| 5 | Rep. agent | irr. | 1.32 | 0.40 | 5.05 | 1.32 | 50 | 97 | 86 | 82 | 98 | 81 | 80 |
| Sensitivity | | | | | | | | | | | | | |
| 2 | Baseline | 0.30 | 1.29 | 0.79 | 3.68 | 1.32 | 82 | 92 | 93 | 81 | 57 | 73 | 80 |
| 2a | $\Omega = .30$ | 0.13 | 1.31 | 0.49 | 4.39 | 1.32 | 78 | 97 | 89 | 82 | 77 | 80 | 80 |
| 2b | $\delta/w = .45$ | 0.81 | 1.29 | 0.77 | 3.69 | 1.32 | 82 | 92 | 93 | 81 | 57 | 74 | 80 |
| 2c | $\delta/w = 2/3$ | 0.07 | 1.29 | 0.79 | 3.68 | 1.32 | 82 | 92 | 93 | 81 | 57 | 73 | 80 |
| 2d | $\kappa = 1$ | 0.39 | 1.29 | 0.96 | 3.44 | 1.32 | 81 | 85 | 94 | 81 | 53 | 64 | 80 |
| 2e | $\beta^I = .9792$ | 0.59 | 1.30 | 0.70 | 3.57 | 1.31 | 90 | 96 | 93 | 82 | 71 | 77 | 80 |
| 2f | $\mu = .173$ | 0.01 | 1.29 | 0.78 | 3.69 | 1.32 | 82 | 92 | 93 | 81 | 57 | 73 | 80 |
| 2g | $\begin{cases} \beta^I = .979 \\ \mu = .173 \end{cases}$ | 0.30 | 1.30 | 0.70 | 3.57 | 1.31 | 90 | 96 | 93 | 82 | 71 | 77 | 80 |
| 2h | $\sigma^{I,P} = .75$ | 0.06 | 1.29 | 0.85 | 3.74 | 1.32 | 78 | 89 | 94 | 81 | 51 | 69 | 80 |
| 2i | $\sigma^{I,P} = 1.5$ | 0.56 | 1.30 | 0.71 | 3.64 | 1.32 | 88 | 95 | 91 | 82 | 65 | 77 | 80 |

*Table 2. Summary Business-Cycle Statistics.*

Note: Models 2, 3, and 5 are linearized and then simulated according to the estimated joint process for $(f_t, s_t, z_t)$. All second-order moments pertain to proportional deviations from the steady state and are thus comparable to the empirical series,which are in log-deviations from trend. Model 4 is simulated as in Krusell and Smith (1998, Sec. IV), except that the shocks have been rescaled so that output deviations from the mean be of the same standard deviation as Model 2 (see Section 3.4.1 and footnote 13 for details).

Table 3 provides additional distributional statistics for the baseline precautionary model. Overall, the wealth share of impatient households displays little aggregate volatility, essentially because its movements are driven by opposing forces (i.e., the intensive versus extensive asset holding margins discussed in Section 2.1). It is also countercyclical because patient households (permanent-income consumers) actively deplete their asset wealth in recessions while employed impatient households (i.e., the precautionary savers) limit the fall in their asset wealth as soon as the job separation rate rises (see equation (EE-I)). For the same reasons, the consumption share of impatient households is not very volatile (when compared to its overall level) and highly procyclical; at a given income process, individual consumption is the mirror image of wealth accumulation. Finally, in our model the share of constrained households tracks the unemployment rate by construction and is thus both volatile and countercyclical.

| | wealth share $A^{I*}/K^*$ | consumption share $C^{I*}/(C^{I*}+C^{P*})$ | share of constrained hh $\Omega(1-n^*)$ |
|---|---|---|---|
| Mean | 0.30 | 40.62 | 3.32 |
| Standard deviation | 0.05 | 0.41 | 0.47 |
| Correlation with output | $-86.29$ | 71.51 | $-83.30$ |

*Table 3. Summary Distributional Statistics.*

### 3.4.3 *Sensitivity*

We now evaluate the robustness of the time-series properties of the precautionary-saving model (Model 2) with respect to changes in the deep parameters of the model. Since we are mainly interested in the determinants of the strength of the precautionary motive, both in the steady state and over the business cycle, we focus exclusively on the parameters that affect the *extent of incomplete insurance* and the *shapes of the cross-sectional distributions of income and wealth.* To this purpose, the bottom part of Table 2 reports the sensitivity of the moments under consideration with respect to changes in the share of impatient households $\Omega$ (Model 2a), the replacement ratio $\delta^I/w^{I*}$ (Models 2b and 2c), the skill premium $\kappa$ (Model 2d), the subjective discount factor of impatient households $\beta^I$ (Models 2e and 2g), the borrowing limit $\mu$ (Models 2f and 2g), and the degree of risk aversion $\sigma^I$ (Models 2h and 2i). The parameters $\Omega$, $\delta^I/w^{I*}$ and $\mu$ directly affect households' ability to insure (or self-insure), while $\beta^I$, $\kappa$ and $\sigma^I$ affect it indirectly via their impact on the equilibrium cross-sectional distributions. Incidentally, we explore the range of admissible parameter values such that our existence conditions (24)–(26) hold.

Unsurprisingly, when $\Omega$ falls, then the population is on average better insured against idiosyncratic shocks, so the dynamics of the model gets closer to that of the representative economy (see Model 2a). More interesting is the impact of the replacement ratio $\delta^I/w^{I*}$ (Models 2b and 2c). Note first that, given the value of the other parameters, the existence of our equilibrium with positive precautionary saving is ensured for $\delta^I/w^{I*} \in [0.42, 0.69]$. When $\delta^I/w^{I*} > 0.69$, households are so well insured that the precautionary motive vanishes and the economy becomes hand-to-mouth (see condition (31)). In contrast, when $\delta^I/w^{I*} < 0.42$ the precautionary motive is so strong, and the implied amount of precautionary wealth so large, that the equilibrium with instant asset liquidation ceases to exist (see condition (26) and condition (33) in the proof of proposition 1).[13] We run our sensitivity experiments with values of $\delta^I/w^{I*}$ that are close to these bounds: $\delta^I/w^{I*} = 0.45$ for Model 2b and $\delta^I/w^{I*} = 2/3$ for Model 2c. We find the second-order moments under consideration to be almost unchanged relative to the baseline case. The only statistics that is significantly altered by the value of $\delta^I/w^{I*}$ is the wealth share of impatient households; this is because, as explained in Section 2.2 above, better direct insurance opportunities crowds out their self-insurance (see Figure 2), thereby deterring the poor to save and raising wealth dispersion.

With Model 2e to 2g, we consider variations in the subjective discount factor ($\beta^I$) and the borrowing limit ($\mu$), both in isolation and jointly. As already mentioned, $\mu$ affects households' ability to smooth idiosyncratic shocks not only directly but also indirectly (via its impact on the distribution of wealth), as does $\beta^I$. The reason for studying joint variations in these two parameters is as follows. From equations (28) and (30), the wealth share $A^{I*}/K^*$ is given by:

$$\frac{A^{I*}}{K^*} = \frac{\Omega\left(a^* - \mu s^*/f^*\right)}{g'^{-1}\left(1/\beta^P - 1 + \nu\right)} = \frac{\Omega\left(n^*\beta^P\left\{u^{I\prime-1}\left[\eta\left(1 + \frac{\beta^P - \beta^I}{\beta^I s^*}\right)\right] - \delta^I - \mu\right\} - \frac{\mu s^*}{f^*}\right)}{g'^{-1}\left(1/\beta^P - 1 + \nu\right)},$$

which is increasing in $\beta^I$ but decreasing in $\mu$. This implicitly defines a set $\left(\mu, \beta^I\right)$ consistent with a given value of $A^{I*}/K^*$ (0.30% in our calibration). Intuitively, the wealth share can be 0.30 either because impatient households have a zero debt limit and the employed are close to the borrowing limit, or because they have a looser borrowing limit but are more patient and hence hold a greater buffer stock ahead of the limit. Let $\mu\left(\beta^I; A^{I*}/K^*\right)$ define the implicit function relating $\mu$ to $\beta^I$ for a given value of $A^{I*}/K^*$, and note from our baseline scenario that $\mu\left(0.972; 0.30\%\right) = 0$. As $\beta^I$ increases, $\mu$ increases (so as to leave $A^{I*}/K^*$ unchanged) and impatient households become more self-insured against unemployment risk (since the size of their buffer ahead of the borrowing limit rises). There is an upper limit to the value of $\beta^I$, above which households are so well self-insured that condition (24) is violated (i.e.,

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[13] See our technical appendix for how to extend the present analysis to the case where full asset liquidation takes more than one period of unemployment.

$c^{eu*} > c^{ue*}$, an inconsistency). Holding the other parameter constant, this value of $\beta^I$ is 0.9792, and the associated debt limit generating $A^{I*}/K^* = 0.3\%$ is $\mu = 0.173$, or 2.15% of average annual labor income.[14] Model 2g studies the impact on the business cycle of the joint change in $\left(\beta^I, \mu\right)$, while Models 2e and 2f each impose one value at a time holding the other parameter at its baseline value.

The cross-sectional distribution of income, and thereby the ability of the different types of households to accumulate assets, is directly affected by the skill premium parameter $\kappa$. In Model 2d we set this parameter to 1, i.e., i.e., we remove any heterogeneity in labor efficiency across patient and impatient households. Since all employed households now earn the same wage, wealth is (slightly) more equally distributed. However, this specification overestimates the consumption of the poorest 60% of the population, as they end up consuming 53% of total consumption (not reported in Table 2) against 41% in the data (see Table 1). Since the consumption of impatient consumers responds more to aggregate shocks than that of patient consumers, the composition effect leads to an overestimation of the volatility of consumption.

Finally, Models 2h and 2i examine how the degree of risk aversion impacts the results. To do this, we assume that $u^P(c) = \left(c^{1-\sigma} - 1\right) / (1 - \sigma)$, $\sigma > 0$, while

$$u^I(c) = \begin{cases} \left(c^{1-\sigma} - 1\right) / (1 - \sigma) & \text{for } c \leq c^* \\ \left(c^{*1-\sigma} - 1\right) + \eta \left(c - c^*\right) & \text{for } c > c^*. \end{cases}$$

Here, $\eta$ is still computed as described in footnote 9 while $c^*$ can be reverse-engineered whenever $c^{eu*} < c^{ue*}$. With such preferences, the parameter $\sigma$ faces similar bounds as for $\delta^I/w^{I*}$: if $\sigma$ is too low, the precautionary motive is so weak that the economy becomes hand-to-mouth, while if $\sigma$ is too high it is so strong that the borrowing constraint is no longer binding for impatient households that become unemployed. Given the other parameters, we find that a steady state with positive precautionary saving requires $\sigma \in [0.70, 1.57]$. We thus compute business cycle statistics for $\sigma = 0.75$ and for $\sigma = 1.5$. As expected, stronger risk aversion leads to more buffer-stock saving by impatient households and hence a greater wealth share $A^{I*}/K^*$. It further leads to a lower consumption volatility and a greater autocorrelation, both of which follow from the fact that intertemporal substituability is lower and hence households wish to smooth consumption more in the face of aggregate shocks.

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[14] Under our calibration the average annual labor income of impatient households is $n^* w^{I*} + (1 - n^*)\,\delta^I = 8.04$.

### 3.4.4 *Comparison with a strictly concave utility function*

While our model is not meant to be an approximation to a fully-fledged heterogeneous-agent model with a strictly concave utility function, it is nevertheless useful to look at the implications of our nonstandard utility function (as opposed to CRRA preferences, for example). We do so by measuring Euler equation errors and proceed as follows. We first solve and run stochastic simulations of our baseline model, and so generate time series for $c_t^{ee}$, $\mathbb{E}_t c_{t+1}^{ee}$, $\mathbb{E}_t c_{t+1}^{eu}$ and $\mathbb{E}_t\left(R_{t+1}\right)$ that are consistent with our quasi-linear utility function.[15] We then use several metrics to measure the quantitative difference between that model and the model with the same dynamics but a strictly concave utility function.

First, we define the *ex ante* interest rate $\Xi_t \equiv \mathbb{E}_t\left(\tilde{R}_{t+1}\right)$ as the interest rate that is consistent with the linearized counterpart of an Euler equation with concave period utility $\tilde{u}\left(c\right) = (c^{1-\tilde{\sigma}} - 1)/(1-\tilde{\sigma})$, $\tilde{\sigma} > 0$:

$$\tilde{u}'\left(c_t^{ee}\right) = \tilde{\beta}\mathbb{E}_t\left[\left[\left(1-s_{t+1}\right)\tilde{u}'\left(c_{t+1}^{ee}\right) + s_{t+1}\tilde{u}'\left(c_{t+1}^{eu}\right)\right]\tilde{R}_{t+1}\right].$$

In Table 4 we report the standard deviation (st$(\cdot)$) as well as the mean absolute value ($\mathbb{E}[|\cdot|]$) of the difference in annualized *ex ante* interest rates, i.e.,

$$\tilde{d}_t \equiv \left(\Xi_t\right)^4 - \left(\mathbb{E}_t\left[R_{t+1}\right]\right)^4.$$

Although we report those statistics for $\tilde{\sigma} = \sigma^I = 1$ (third row of Table 4), we emphasize here that our baseline utility function (36) is *not* comparable to a log utility over $[0, +\infty)$; this is because our economy displays significantly more intertemporal substituability (due to the linear portion of the utility function) than an economy with a strictly concave utility function over $[0, +\infty)$. This leads us to compute $\tilde{d}_t$ for lower values of $\tilde{\sigma}$, holding $\sigma^I = 1$ unchanged. In all these experiments we adjust the subjective discount factor $\tilde{\beta}$ so that the mean of $\Xi_t$ is the same as that generated by our model, $R^*$. That is, we set:

$$\tilde{\beta} = \beta^I\tilde{u}'\left(c^{ee*}\right)/\left[\left(1-s^*\right)\tilde{u}'\left(c^{ee*}\right) + s^*\tilde{u}'\left(c^{eu*}\right)\right],$$

and report the corresponding value of $\tilde{\beta}$ in Table 4 (second column).

Finally, for all the cases under study we also report Euler equation errors in terms of proportional consumption difference, following Judd (1992). More specifically, using the interest rate generated by our baseline model (i.e., $R_{t+1}$), we compute the current consumption

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[15] The expectations $\mathbb{E}_t(c_{t+1}^{ej})$, $j = e, u$, and $\mathbb{E}_t\left(R_{t+1}\right)$ are computed using the state-space representation of the model, $\mathbf{z}_t = \mathbf{B}\mathbf{z}_{t-1} + \mathbf{x}_t$ (see Section 4.4).

level consistent with the following Euler equation:

$$\left(\tilde{c}_t^{ee}\right)^{-\tilde{\sigma}} = \tilde{\beta}\mathbb{E}_t\left[\left[\left(1-s_{t+1}\right)\left(c_{t+1}^{ee}\right)^{-\tilde{\sigma}} + s'_{t+1}\left(c_{t+1}^{eu}\right)^{-\tilde{\sigma}}\right]R_{t+1}\right],$$

where $\tilde{c}_t^{ee}$ is the implicit consumption level implied by a strictly concave utility function. We then define

$$\tilde{e}_t \equiv \left(\tilde{c}_t^{ee} - c_t^{ee}\right)/c_t^{ee}$$

and report $\text{st}[|\tilde{e}_t|]$ and $\mathbb{E}[|\tilde{e}_t|]$ in the last two columns of Table 4. Note that the value of $\tilde{\beta}$ that equates $\mathbb{E}\left(\Xi_t\right)$ and $R^*$ also equates $\mathbb{E}\left[\tilde{c}_t^{ee}\right]$ with $c^{ee*}$.

| *Specification* | | | *Statistics (%)* | | |
|---|---|---|---|---|---|
| | $\tilde{\beta}$ | $\text{st}(\tilde{d}_t)$ | $\mathbb{E}[\lvert\tilde{d}_t\rvert]$ | $\text{st}\left(\tilde{e}_t\right)$ | $\mathbb{E}[\lvert\tilde{e}_t\rvert]$ |
| $\tilde{\sigma} = 1$ | 0.972 | 4.73 | 3.77 | 1.14 | 0.91 |
| $\tilde{\sigma} = 1/3$ | 0.985 | 1.64 | 1.31 | 1.18 | 0.94 |
| $\tilde{\sigma} = 0.1$ | 0.988 | 0.58 | 0.46 | 1.40 | 1.12 |

*Table 4. Euler Equation Errors*

Table 4 shows that Euler equation errors, when measured as proportional consumption differences, are of the order of 1% and do not exceed 1.40%. When measured as differences in the annualized interest rate, the distance between the Euler equations strongly depends on the amount of curvature of the strictly concave utility function $\tilde{u}(\cdot)$. For example, for $\tilde{\sigma} = \sigma^I = 1$ the difference in the annualized interest has a standard deviation of 6.30%, which is large. However, as explained above in this situation the utility function with a linear part in fact generates much more intertemporal substituability than a log utility function over $[0, +\infty)$; hence a meaningful comparison between the two utility functions can only be made for lower values of $\tilde{\sigma}$, in which case the volatility of the interest rate difference is not necessarily large (e.g., 0.58% for $\tilde{\sigma} = 0.1$).

# 4 Concluding Remarks

In this paper, we have proposed a tractable general equilibrium model of households' behavior under incomplete insurance and time-varying precautionary savings. We have further gauged its ability to shed light on the dynamics of aggregate consumption over the business cycle. In contrast to earlier attempts at constructing tractable versions of models with heterogeneous agents, ours has two specific features. First, we provide a realistic representation of households' labor income risk, resulting from the combined effects of persistent changes in the equilibrium real wage and the probability of changing employment status. Second, we were able to reduce the model's dynamics to a small-scale system solved under rational expectations, thanks to exact cross-household aggregation. We calibrate the model to match the broad features of the cross-sectional wealth and consumption dispersions that are observed in the US economy. We then feed it with joint productivity and labor market shocks. Despite its simplicity the model does a fairly reasonable job at explaining the time-series behavior of aggregate consumption. In particular, comparison with the pure "hand-to-mouth" model reveals that time-variations in precautionary savings may significantly raise consumption volatility, even though the average individual wealth of precautionary savers (as a share of aggregate liquid wealth) is very low. The flip side of the coin concerns the behavior of aggregate investment: because in our model the precautionary motive is high during recessions and low during booms, the economy with time-varying precautionary saving tends to display less investment volatility than the representative-agent model. Smoother investment then translates in smoother capital and output, so the latter turns out to be less volatile in our baseline economy than in its representative-agent counterpart. This property, which we share with Krusell and Smith (1998), suggests that incomplete insurance must be interacted with other frictions to raise *output* volatility (relative to comparable complete-insurance economies), in addition to raising consumption volatility.

There are several directions of research that our tractable framework will allow us to explore. A first natural direction is the consideration of other frictions (in addition to incomplete insurance) that are widely believed to matter for the amplification and propagation of business cycle shocks–notably labour-market search and nominal rigidities. The key question here is whether or not those additional frictions can improve on the main limitation of the basic single-friction model: the fact that in this model time variations in precautionary savings exert a stabilizing force on investment fluctuations. Another area where this framework can be applied is that of international interdependences. It is well known that in an open economy context incomplete insurance against idiosyncratic shocks affects both trade and the pattern of capital flows. Indeed, the demand for both foreign goods and foreign assets is affected by precautionary savings. Our approach opens the way to the construction of a multi-country analysis wherein the interactions between aggregate and idiosyncratic income shocks can be investigated.

*CNRS, Ecole Polytechnique and Banque de France*
*CNRS, Paris School of Economics and OFCE*

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# References

[1] Acemoglu, D. and Autor, D.H. (2011), 'Skills, tasks and technologies: Implications for employment and earnings', in O. Ashenfelter and D. Card (eds.), *Handbook of Labor Economics*, vol. 4B, Elsevier B.V., pp. 1043-1171.

[2] Aiyagari, S. R. (1994), 'Uninsured idiosyncratic risk and aggregate saving', *Quarterly Journal of Economics*, vol. 109, pp. 659-684.

[3] Becker, R.A. (1980), 'On the long-run steady state in a simple dynamic model of equilibrium with heterogenous households', *Quarterly Journal of Economics*, vol. 95, pp. 375-382.

[4] Becker, R.A. and Foias, C. (1987), 'A characterization of Ramsey equilibrium', *Journal of Economic Theory*, vol. 41, pp. 173-184.

[5] Braun, R.A and Nakajima, T. (2012), 'Uninsured countercyclical risk: an aggregation result and application to optimal monetary policy', *Journal of the European Economic Association*, vol. 10(6), pp. 1450-1474.

[6] Campbell, J.Y. and Mankiw, N.G. (1989), 'Consumption, income and interest rates: reinterpreting the time series evidence', *NBER Macroeconomics Annuals*, vol. 11, pp. 155-201.

[7] Carroll, C.D. (1994), 'How does future income affect current consumption?', *Quarterly Journal of Economics*, vol. 109(1), pp. 111-147.

[8] Carroll, C.D. and Samwick, A. A. (1998), 'How important is precautionary saving?', *Review of Economics and Statistics*, vol. 80(3), pp. 410-419.

[9] Carroll, C.D. and Samwick, A. A. (1997), 'The nature of precautionary wealth', *Journal of Monetary Economics*, vol. 40(1), pp. 41–71.

[10] Carroll, C.D., Slacalek, J. and Sommer, M. (2012), 'Dissecting saving dynamics: measuring credit, wealth and precautionary effects', IMF Working Paper 12/219.

[11] Carroll, C.D. and Toche, P. (2011), 'A tractable model of buffer stock saving', Working Paper.

[12] Chetty, R. (2008), 'Moral hazard versus liquidity and optimal unemployment insurance', *Journal of Political Economy*, vol. 116(2), pp. 173-234.

[13] Cochrane, J. (1991), 'A simple test of consumption insurance', *Journal of Political Economy*, vol. 99, pp. 957-976

[14] Constantinides, G. and Duffie, D. (1996), 'Asset pricing with heterogenous consumers', *Journal of Political Economy*, vol. 104(2), pp. 219-240.

[15] Engen, E.M. and Gruber, J. (2001), 'Unemployment insurance and precautionary saving', *Journal of Monetary Economics*, vol. 47(3), pp. 545-579.

[16] Gali, J., Lopez-Salido, J.D. and Valles, J. (2007), 'Understanding the effects of government spending on consumption', *Journal of the European Economic Association*, vol. 5(1), pp. 227-270.

[17] Gourinchas, P.-O., Parker J., (2001). 'The empirical importance of precautionary saving', *American Economic Review*, vol. 91(2), pp. 406-412

[18] Gruber, J. (1997), 'The consumption smoothing benefit of unemployment insurance', *American Economic Review*, vol. 87, pp. 192-205.

[19] Gruber, J. (1998), 'Unemployment insurance, consumption smoothing, and private insurance: Evidence from the PSID and CEX', in Research in Employment Policy, vol. 1, L.J. Bassi and S.A. Woodbury (eds), Jai Press, Stamford, CT.

[20] Guerrieri, V. and Lorenzoni, G. (2011), 'Credit crises, precautionary savings, and the liquidity trap', NBER Working Papers 17583.

[21] Guerrieri, V. and Lorenzoni, G. (2009), 'Liquidity and trading dynamics', *Econometrica*, vol. 77(6), pp. 1751-1790.

[22] Heathcote, J., Perri, F. and Violante, G. (2010), 'Unequal we stand: An empirical analysis of economic inequality in the United States 1967-2006', *Review of Economic Dynamics*, vol. 13(1), pp. 15-51.

[23] Heathcote, J., Storesletten, K. and Violante, G.L. (2013), 'Consumption and labour supply with partial insurance: An analytical approach', *American Economic Review*, forthcoming.

[24] Heathcote, J., Storesletten, K. and Violante, G.L. (2013), 'Insurance and opportunities: A welfare analysis of labor market risk', *Journal of Monetary Economics*, vol. 55, pp. 501-525.

[25] Iacoviello, M. (2005), ‘House prices, borrowing constraints, and monetary policy in the business cycle’, *American Economic Review*, vol. 95(3), pp. 739-764.

[26] Judd, K.L. (1992), ‘Projection methods for solving aggregate growth models’, *Journal of Economic Theory*, vol. 58, pp. 410-452.

[27] Kaplan, G. and Violante, G.L. (2012), ‘A model of the consumption response to fiscal stimulus payments’, Working Paper.

[28] Kimball, M.S. (1990), ‘Precautionary saving in the small and in the large’, *Econometrica*, vol. 58(1), pp. 53-73

[29] Kiyotaki, N. and Moore, J. (1997), ‘Credit cycles’, *Journal of Political Economy*, vol. 105(2), pp. 211-248.

[30] Krusell, P., Mukoyama, T. and Sahin, A. (2011), ‘Labor-market matching with precautionary savings and aggregate fluctuations’, *Review of Economic Studies*, vol. 77 (4), pp. 1477-1507.

[31] Krusell, P. and Smith, A.A. (1998), ‘Income and wealth heterogeneity in the macroeconomy’, *Journal of Political Economy*, vol. 106(5), pp. 867-896.

[32] Lagos, R. and Wright, R. (2005), ‘A unified framework for monetary theory and policy analysis’, *Journal of Political Economy*, vol. 113, 463-484.

[33] McKay, A. and Reis, R. (2013), ‘The role of automatic stabilizers in the U.S. business cycle’, Working Paper.

[34] Mengus, E. and Pancrazi, R. (2012), ‘Participation costs and consumption smoothing: Theory and empirical evidence’, Working Paper.

[35] Mertens, K. and Ravn, M. (2011), ‘Understanding the aggregate effects of anticipated and unanticipated tax policy shocks’, *Review of Economic Dynamics*, vol. 14(1), pp. 27-54.

[36] Merz, M. (1995), ‘Search in the labor market and the real business cycle’, *Journal of Monetary Economics*, vol. 36(2), pp. 269–300.

[37] Parker, J.A. and Preston, B. (2005), ‘Precautionary saving and consumption fluctuations’, *American Economic Review*, vol. 95(4), pp. 1119-1143.

[38] Ravn, M. and Sterk, V. (2013), 'Job uncertainty and deep recessions', Working Paper.

[39] Shimer, R. (2005), 'The cyclical behaviour of equilibrium employment and vacancies', *American Economic Review*, vol. 95(1), pp. 25–49.

[40] Toche (2005), 'A tractable model of precautionary saving in continuous time,' *Economics Letters*, vol. 87(2), 267–272.

[41] Vermylen, K. (2006), 'Heterogenous agents and uninsurable idiosyncratic employment shocks in a linearized dynamic general equilibrium model', *Journal of Money, Credit and Banking*, vol. 38(3), 837–838.