Sovereign Default and Liquidity: The Case for a World Safe Asset*

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Abstract

This paper presents a positive and normative study of a world financial market when sovereign countries can default on their debt. In our model, the world interest rate is determined by countries' net savings and default decision. The amount of safe assets is endogenous and determines international risk-sharing. First, we show that non-trivial multiple equilibria naturally arise, due to the endogeneity of the interest rate. The quantity of safe assets is thus not uniquely determined. Second, even the market equilibrium with the highest welfare is not constrained-efficient because the market incentives for defaulting are too high. Third, we prove that a world fund issuing a safe asset increases aggregate welfare. Its relationship with the IMF's Special Drawing Rights is discussed.

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1 Introduction

The global economy exhibits two related features. The first is the pervasiveness of sovereign default. The largest default in history (by present value) was the 2012 Greek debt restructuring, which covered more than €200 bn of privately held debt (Tomz and Wright 2013). Current debates regarding a larger restructuring of Greek debt or the default of another developed country indicate that sovereign default may reach another order of magnitude in the near future.

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Even if the default risk fails to materialize, the fact that it is discussed is a clear indication that the perceived safeness of certain countries' public debt has deteriorated. The second feature affecting the global economy is the apparent shortage of safe assets, which explains the downward trend in the return on US public debt.[1] These two facts are obviously related, as sovereign default almost mechanically reduces the quantity of safe assets. In addition, the quantity of safe assets affects real interest rate levels and the default probability through its effect on both the incentives to accumulate assets and the opportunity cost of default.

The goal of this paper is to investigate the interactions between the quantity of safe assets and sovereign default in general equilibrium. We are interested in both positive and normative questions. What are the determinants of the quantity of safe assets when sovereigns can choose to strategically default? Does the market economy generate too many defaults as a general equilibrium outcome? Should a world institution issue a safe asset? If so, how much should it issue? This last question was, for instance, discussed in policy circles at the IMF before the decision to issue Special Drawing Rights.

To help answer these questions, this paper presents a general equilibrium model of default where sovereigns can borrow or lend on international markets, and possibly default on their debt, in order to smooth idiosyncratic income shocks. Elaborating on the Eaton and Gersovitz (1981) literature, the basic friction we consider is the lack of a complete insurance market for country-specific risk. Countries can only issue non-contingent claims, on which they can default. Equilibrium default can occur when the debt burden is high in bad times and when the opportunity cost of default is low. Following the work of Aguiar and Gopinath (2006) and Arellano (2008), among others, this framework has become a benchmark for studying sovereign default (see the recent paper by Amador and Aguiar 2015). In this literature, the authors assume that the riskless real interest rate is exogenous, which enables them to investigate the amount of debt and the default decision in rich environments (see the literature review below). Our contribution is to endogenize the world interest rate by developing a tractable general equilibrium model and to derive welfare analysis.

We first model the world economy as a continuum of countries borrowing and lending to one other to smooth idiosyncratic shocks. Countries can default on their debt depending on

[1] Caballero and Farhi (2017), Barro and Mollerus (2016), and Hall (2016), among others, discuss the effects of a shortage of safe assets. See also Gorton, Lewellen, and Metrick (2012) for a measure of the safe asset share in the US economy.

the endogenous intertemporal cost of default. When countries default, they are temporarily excluded from the financial markets. The world interest rate is determined by the international financial market clearing system. Equilibrium in this economy is characterized by the default policy, world wealth distribution, and world interest rates. An alternative way of considering this economy is to think about a Bewley economy where agents face no exogenous credit constraint but may default if they find it optimal to do so. Surprisingly, the constrained-efficiency of market economies and of optimal policies in these environments has yet to be studied. To do so, we use the strategy of Davila, Hong, Krusell, and Rios-Rull (2012), developed in incomplete-market economies without default, in order to derive the constrained-efficient allocation. We then analyze the properties of the market economy, in particular welfare and the quantity of safe assets determining international risk-sharing. We derive three main results.

The first result is that the interaction between incomplete markets and default generates multiple equilibria, which differ in terms of the quantity of safe assets and interest rates. When the interest rate is high, the incentives to save for self-insurance are high. As countries save more, there are fewer defaults because the opportunity cost of defaulting is high. Participating in financial markets is highly valued as risk-sharing is important. A high supply of safe assets is consistent with a low default risk. Conversely, when a smaller supply of safe assets is available for self-insurance, there is a higher default rate in general equilibrium, as the relative gain of participating in financial markets is low. The multiple equilibria can be ranked according to their aggregate welfare. An equilibrium with fewer defaults unambiguously corresponds to higher welfare, compared to an equilibrium with a higher number of defaults. The multiplicity of equilibria thus crucially depends on the endogeneity of the interest rate.

Second, we show that the market equilibrium with the highest welfare is still constrained-inefficient. The reason being that when markets are incomplete, prices do not convey the right incentives to save. In other words, there is a pecuniary externality, implying a lack of risk-sharing and a shortage of safe assets in the market economy. Sovereigns do not properly internalize the social benefits of their debt as a safe asset for other countries in their net saving and default decisions, as markets are incomplete.

Third, we prove that introducing an International Financial Institution (IFI) increases aggregate welfare. The IFI issues interest-bearing assets that are financed by the voluntary contributions of member states. The size of the IFI depends on the assumptions about its commitment devices. First, if the IFI has a strong commitment device, it can implement full risk-sharing among member states, but default still occurs in general equilibrium. This is a property of the constrained-efficient allocation. In this case, the IFI must be able to limit the total liquidity held by any country. Second, if the IFI can issue liquidity but cannot interfere with the countries' saving decisions, risk-sharing among countries can be improved but full risk-sharing among IFI's members cannot be reached. We calibrate the model using data on sovereign default and "disaster events" to reproduce both the default probability and income fluctuations. For the realistic case of imperfect risk-sharing, the optimal quantity of assets issued by the IFI represents around 9% of world GDP. Countries' average contributions are equal to 0.39% of their own GDP.

As a final step, we verify that these results are robust to the introduction of a large country – with positive mass – internalizing the effect of its saving decisions on the world financial market. Following Farhi and Maggiori (2018), we call this country the *Hegemon*. We prove that the main results hold, even in the presence of the Hegemon. The introduction of an IFI still improves aggregate welfare by issuing outside liquidity.

The IFI and its asset supply are obviously reminiscent of the IMF Special Drawing Rights (SDRs). These SDRs were issued in the early 1970s after a global discussion about the scarcity of safe stores of value (see Williamson 2009 for a short history). SDRs are interest-bearing assets, whose interest rate is determined weekly as the average interest rate on the money markets for a basket of currencies.[2] There are nevertheless two main differences between the assets issued by the IFI in our model and SDRs. First, the interest rate on the assets issued by the IFI in our model should be linked to the interest paid by governments issuing safe assets rather than the interest on money markets, as is the case for SDRs. Our safe asset is not a currency but a remunerated store of value. Second, the outstanding amounts differ substantially. For SDRs, the outstanding amount was smaller than 0.3% of world GDP in 2016, while, with our calibration, the optimal issuance of world safe assets should correspond to 9% of world GDP.

Literature review. This model provides positive and normative implications for sovereign default in general equilibrium. It follows the tradition of Eaton and Gersovitz (1981) based on incomplete insurance markets, which have been shown to be quantitatively relevant (Aguiar and

[2]See the IMF website for a precise definition and SDR interest rate values: `http://www.imf.org/external/np/fin/data/sdr_ir.aspx`.

Gopinath 2006, Arellano 2008, among others, as well as the recent survey of Amador and Aguiar 2015). The same tools (incomplete insurance markets and strategic default) have been used to study household default in general equilibrium. Chatterjee, Cordae, Nakajima, and Rios-Rull (2007) and Livshits, MacGee, and Tertilt (2007) use this model to compare the welfare effect of different legal frameworks on household default. Instead, we provide a tractable model to characterize the constrained efficient equilibria using the methodology of Davila, Hong, Krusell, and Rios-Rull (2012) and provide a mechanism to decentralize it. Tractability comes from the use of a quasi-linear utility function, which is known to help reducing the state space in incomplete-market economies, as in Scheinkman and Weiss (1986), Lagos and Wright (2005) or Challe, Le Grand, and Ragot (2013).

Our analysis stresses the role of the quantity of assets required to self-insure against idiosyncratic shocks. It is thus related to the recent literature on safe assets, in particular the literature focusing on the quantity of the risk-adjusted store of value provided by the market economy. Authors (Gorton, Lewellen, and Metrick 2012, Barro and Mollerus 2016, Hall 2016, and Caballero and Farhi 2017, among others) have studied the implications of a possible lack of safe assets. Another large body of literature investigates the determinants of the quantity of world safe assets and of the interest rate (Mendoza, Quadrini, and Rios-Rull 2009, Azzimonti, de Francesco, and Quadrini 2014) and the decisions of sovereigns to strategically choose their level of public debt with roll-over risk (He, Krishnamurty, and Milbradt 2019). Farhi and Maggiori (2018) propose a model of the international monetary system and study various market structures and frictions. We instead identify the distortions on the world financial markets in the absence of these strategic behaviors.

The source of multiple equilibria is new to the best of our knowledge, although other mechanisms generating equilibrium multiplicity have been discussed in the literature. First, a large body of literature has emphasized roll-over risk, following Calvo (1988) or Cole and Kehoe (2000). Recent references include Lorenzoni and Werning (2013) and He, Krishnamurty, and Milbradt (2019). In our model, this type of risk does not exist. Recently, Arellano, Bai, and Lizarazo (2018) have proved the presence of multiple equilibria generated by negative wealth shocks on risk-adverse lenders. Our mechanism for equilibrium multiplicity is different. It is based on the endogeneity of the quantity of safe assets, which affects both the cost of self-insurance and the incentives to default. Indeed, when the interest rate is exogenous, the equilibrium is unique in our Eaton-Gersovitz environment as shown by Auclert and Rognlie (2016).[3]

Section 2 presents the environment. Section 3 solves the model for the market equilibrium. Section 4 presents a special case of the model to identify all mechanisms generating equilibrium multiplicity and a role for international liquidity. Section 5 presents the general case for equilibrium multiplicity and constrained-efficient allocation. Section 6 also shows that introducing an IFI issuing outside liquidity improves risk-sharing and increases aggregate welfare. Section 7 presents a numerical application. Section 8 concludes.

2 Environment

2.1 Set-up

Time is discrete $t = 1, \ldots, \infty$. We first model the world economy as a continuum of small open economies. Countries are distributed in each period t according to a uniform distribution G over a segment I of length 1, without mass points.[4] All countries are thus small and have no market power in the world financial market. We discuss below the case where be a large country has a financial market power, to verify the generality of our results.

Each economy has a risky production technology and a benevolent government maximizes utility on behalf of a unit mass of identical consumers. Countries face idiosyncratic production shocks and there is no aggregate shock at the world level.

Each country has identical, additive, and time-separable preferences over streams of consumption $(c_t)_{t\geq 0}$ and labor supply $(l_t)_{t\geq 0}$. The period utility function over consumption c and labor supply l is $u(c) - l$, where the consumption utility function u is increasing, twice continuously differentiable, and concave. In the applications below, u will be CRRA: $u(c) = (c^{1-\sigma} - 1) / (1-\sigma)$ if $\sigma \neq 1$ and $\mathrm{u}(c) = \ln(c)$ otherwise. The micro-foundation for this linear disutility of labor is the existence of both indivisible labor and a complete market within the country as shown by Hansen (1985) and Rogerson (1988).[5] In each country, the government

[3] Finally, there is a different literature stream on default in general equilibrium with complete insurance markets, where default generates utility costs. See, for instance, Dubey, Geanakoplos, and Shubik (2005), among others.

[4] This representation of the world economies follows Clarida (1990) or Bai and Zhang (2010). There is a body of literature examining the applicability of the law of large numbers in continuum economies – see Feldman and Gilles (1985) and Green (1994), among others. Here, we simply assume that the law of large numbers applies.

[5] The utility function is used by Lagos and Wright (2005) in a matching environment and by Scheinkman and Weiss (1986) and Challe, LeGrand, and Ragot (2013), among others, in an incomplete-market environment without default. It simplifies the analysis, as shown below.

maximizes the intertemporal welfare $\sum_{t=0}^{\infty}\beta^t (u(c_t) - l_t)$, where $\beta \in (0,1)$ is the constant discount factor.

Countries face a productivity risk that can be neither avoided nor insured. The productivity status of a given country can be in one of two states, which are described as *productive* (state p) and *unproductive* (state u). When a country is productive, it has access to a linear production technology, which transforms l units of labor into l units of final goods. The country's labor supply can, in addition, be freely adjusted in every period. When a country is unproductive, it is restricted to supplying an amount $\bar{l} < 1$ of labor. This restriction implies that the marginal utility of consumption for a country in a productive state is smaller than that of a country in an unproductive state.

The productivity status follows a first-order Markov chain with the transition matrix $\Pi = \begin{bmatrix} \alpha & 1-\alpha \\ 1-\rho & \rho \end{bmatrix} \in (0,1)^4$, where the probability $1-\alpha$ is the probability of switching from state p in the current period to state u in the next one, for example. The stationary distribution of this process implies that the share of productive countries is $n^p = (1-\rho)/(2-\alpha-\rho)$, and the share of unproductive countries is $1 - n^p$.

2.2 Financial markets and default

As is standard in the literature on sovereign default, following Eaton and Gersovitz (1981) and Bulow and Rogoff (1989), international financial markets are assumed to be plagued by two frictions. First, markets are incomplete for the country-specific production shock. Second, countries can default on their debt, but at the cost of being temporarily excluded from the financial markets. We assume that the current productivity status and the asset volume for each country are observable when debt is traded on international markets. If a country defaults on its debt, it is excluded from international markets and has to live in a state of autarky before potentially rejoining the international financial markets. Excluded countries are still affected by the productivity shock.

To make the model empirically relevant, we follow Arellano (2008) and introduce an additional cost of default. Countries in autarky only have access to a less efficient production technology, through which 1 unit of labor is transformed into only $\varphi \in (0,1]$ units of consumption. Excluded productive countries have the probability $\theta \in (0,1)$ of reentering the financial markets in every period, while excluded unproductive agents cannot reenter the financial markets. When reentering the financial markets, countries are endowed with zero financial wealth.

Debt is traded through a unit mass of competing risk-neutral financial intermediaries. Saving countries lend to intermediaries and borrowing countries borrow from them. Financial intermediaries diversify their risk across countries and act as devices for pooling idiosyncratic risk. Since there is no aggregate risk, financial intermediaries can charge a credit risk premium to borrowers according to their default probability and savers can save in a safe asset.

We denote by $q\left(B', s\right)$ the price of a claim on one unit of a next period good for a country that chooses an amount of asset B' and that has the production status $s = \{p, u\}$. If $B' > 0$, the country is saving in a safe asset of price $q\left(B', s\right) = q$. The real interest rate on the safe asset is simply $1 + r \equiv \frac{1}{q}$. If $B' < 0$, the country is borrowing. We follow Arellano, Bai, and Bocola (2017) and Arellano, Bai, and Mihalache (2019) and assume that the debt repayment in cases of default is exogenous and equal to $\underline{B} < 0$. The recovery rate is therefore endogenous and amounts to $\underline{B}/B'$, which depends on the asset choice B'. This recovery rate is introduced to be consistent with the data, but is not important for the qualitative results.

Since intermediaries are perfectly diversified, the price $q(B', s)$ can be expressed as:

$$q\left(B', s\right) = q \times \left(1 - \delta\left(B', s\right) + \delta\left(B', s\right) \frac{\underline{B}}{B'}\right),$$

where $\delta\left(B', s\right)$ is the default probability of a country choosing a debt B' while its current status is $s \in \{p, u\}$. Importantly, both the price of a safe asset q and the default probability $\delta\left(B', s\right)$ will be general equilibrium endogenous outcomes.

A country in state $s \in \{p, u\}$, endowed with the beginning-of-period wealth B, trading an amount of debt B', and supplying a labor quantity l, will have consumption equal to:

$$c = l + B - q\left(B', s\right) B'. \tag{1}$$

If the country is in a productive state, it can then freely adjust its labor effort l. If unproductive, the country will supply the fixed amount $l = \bar{l}$.

Since the default probability $\delta\left(B', s\right)$ is not necessarily continuous for the debt level B', the budget set may not be convex. However, although the problem is not convex, it can be written in recursive form (see Stokey and Lucas 1989, Theorem 9.4 and Auclert and Rognlie 2016 for the properties of the equilibrium structure), which allows us to simplify the exposition and to derive the necessary first-order conditions.

2.3 Welfare functions and financial market clearing

We define $V_s^o(B)$ as the value function of a country that participates in the financial markets, that starts the current period with the asset holding B, and that has the productivity status $s \in \{p, u\}$.[6] The superscript o denotes a participating country that has the *option* to default on B. The country then decides whether to default or repay its debts by choosing the option associated with the highest welfare. We denote by $V_s^c(B)$ the value function of a country deciding to repay the debt B, while its status is s. The superscript c indicates that the country decides to *continue* to honor its debts. Similarly, the value function $V_s^d(b)$ is the value function of a country that decides to *default*. The dependence in b is only designed to take into account the fixed debt repayment $\underline{B}$, in a recursive formula, as can be seen in equations (3) and (4) below. . The value function $V_s^o(B)$ is thus equal to the maximum between the value functions associated with debt repayment or default. Formally:

$$\forall B, \forall s \in \{p, u\}, \quad V_s^o(B) = \max\{V_s^c(B), V_s^d(\underline{B})\}. \tag{2}$$

Let us now turn to the expression of value functions V^d and V^c. Since the probability of defaulting countries reentering the financial markets when productive with zero wealth is θ, the value function associated with default can be expressed as:

$$V_p^d(b) = \max_l \ (u(\varphi l + b) - l) + \beta\alpha\left(\theta V_p^o(0) + (1-\theta) V_p^d(0)\right) + \beta(1-\alpha)V_u^d(0), \tag{3}$$

$$V_u^d(b) = u\left(\varphi\bar{l} + b\right) - \bar{l} + \beta(1-\rho)\left(\theta V_p^o(0) + (1-\theta) V_p^d(0)\right) + \beta\rho V_u^d(0). \tag{4}$$

In these two equations, the country is excluded from the financial markets and trades no assets. The whole production φl or $\varphi\bar{l}$ of a period is consumed within the same period. The dependence in b in (3) and (4) is purely designed to distinguish between countries that have just defaulted and are paying the debt amount $\underline{B}$ and countries that were already in autarky.

When a country participates in the financial markets and chooses not to default in the current

[6] For the sake of simplicity, we follow the notations of Arellano (2008).

period, the value function V^c can be expressed as follows, for any debt endowment B:

$$V_p^c(B) = \max_{l,B'} \; u(l + B - q(B', s) B') - l + \beta\left(\alpha V_p^o(B') + (1-\alpha) V_u^o(B')\right), \tag{5}$$

$$V_u^c(B) = \max_{B'} \; u(\bar{l} + B - q(B', s) B') - \bar{l} + \beta\left((1-\rho) V_p^o(B') + \rho V_u^o(B')\right), \tag{6}$$

where we use the budget constraint (1) to substitute for the consumption expression. As the value functions V_s^c for $s = p, u$ are increasing in the wealth amount B, we can define the quantities $\overline{B}^s \in \mathbb{R} \cup \{-\infty\}$, $s = p, u$ as:

$$\overline{B}^s = \min\{B \in \mathbb{R} | V_s^c(B) \geq V_s^d(\underline{B})\}, s = p, u. \tag{7}$$

In other words, the quantity $\overline{B}^s$ is the state-contingent threshold such that a country endowed with a wealth level above the threshold will decide not to default.

Finally, we provide the expression for financial market clearing. Since assets are in zero net supply and competition among financial intermediaries implies a zero-profit condition, the value of all net asset demands sums to zero. If we recall that countries are distributed along the segment I with the distribution G, financial market clearing can be formalized as follows:

$$\int_{i \in I} q\left(B'^i, s^i\right) B'^i G(di) = 0, \tag{8}$$

where B'^i is the asset demand of country $i \in I$ in the current period. We focus on the steady state equilibrium where the price of the safe asset q is constant. We provide a more formal definition of the equilibrium.

Definition 1 (Competitive equilibrium) *A competitive equilibrium is a price q, risk premia $(\delta(B, s))_{s=u,p,B\in\mathbb{R}}$, policy functions $(B'(B, s))_{s=u,p,B\in\mathbb{R}}$, $l(B, p)_{B\in\mathbb{R}}$, and the level of production in autarky l^d such that: 1) for a given price q and given risk premia $(\delta(B, s))_{s=u,p,B\in\mathbb{R}}$, policy functions solve the program (2)–(6); 2) the risk premia are consistent with perfect competition and full diversification for financial intermediaries; and 3) the price q is such that the financial market clears, i.e., equation (8) holds.*

3 Market equilibrium

We present the main properties of the market equilibrium. We start by the equilibrium price schedule, which is standard in this type of environment. Following Arellano (2008), among others, we know that two asset thresholds $\overline{B}^e$and $\overline{B}^u$, $\overline{B}^e < \overline{B}^u < 0$ exist such that:

$$q(B') = \begin{cases} q & \text{if } B' \geq \overline{B}^u, \\ q\left(\alpha + (1-\alpha)\frac{B}{B'}\right) & \text{if } \overline{B}^e \leq B' < \overline{B}^u, \\ q\frac{B}{B'} & \text{if } B' < \overline{B}^e, \end{cases} \text{ and } q(B', u) = \begin{cases} q & \text{if } B' \geq \overline{B}^u, \\ q\left(1-\rho+\rho\frac{B}{B'}\right) & \text{if } \overline{B}^e \leq B' < \overline{B}^u, \\ q\frac{B}{B'} & \text{if } B' < \overline{B}^e. \end{cases} \tag{9}$$

Equation (9) states that the price schedule is a step function with two thresholds $\overline{B}^e$ and $\overline{B}^u$, where $\overline{B}^e < \overline{B}^u$. If a country's net saving is higher than $\overline{B}^u$, then it will not default in the next period, regardless of its idiosyncratic shock. In this case, the price of its claim is the price of a safe asset q. If the country saves less than $\overline{B}^u$ but more than a second threshold $\overline{B}^e$, it will default in cases of bad idiosyncratic shock, but not in cases of a good shock. Consequently, the price of the claim is the price q multiplied by the expected payoff in the next period. This expected payoff is either 1 if the country remains productive or the recovery ratio $\frac{B}{B'}$ if the country becomes unproductive and defaults. Finally, for net savings that are strictly smaller than $\overline{B}^e$, the country defaults regardless of the next-period idiosyncratic shock, so that the price of the claim is $q\frac{B}{B'}$.

3.1 Euler equations and budget constraints

Quasi-linearity in the labor supply considerably simplifies the analysis of the equilibrium. To begin with, the first-order condition for the labor supply l of productive countries in (5) yields $u'(c_0) = 1$. As a result, we find that the consumption level of productive countries is $c_0 = u'^{-1}(1)$. Using the budget constraint (1), the value function (5) becomes:

$$V^c(B, p) = V^c(0, p) + B. \tag{10}$$

The value function of productive countries participating in the financial markets is thus linear in the beginning-of-period asset endowment. All productive non-defaulting countries therefore have the same marginal utility, which is independent of their beginning-of-period endowment.

We now prove that the equilibrium is characterized by a sequence $\{B_k\}_{k=0,\ldots,\infty}$, such that all productive countries participating in financial markets save B_0 while participating countries that are unproductive for $k \geq 1$ consecutive periods save the same amount B_k. In equilibrium, the saving (or borrowing) decision of agents thus simply depends on their idiosyncratic production history, which considerably simplifies aggregation.

We adopt a guess-and-verify strategy to simplify the exposition. Assume that countries default after being unproductive for $D \leq \infty$ consecutive periods, where the case $D = \infty$ corresponds to countries never defaulting. The first-order conditions for asset choices are:

$$q = \beta \left(\alpha + (1-\alpha) u'\left(c_1\right)\right), \tag{11}$$

$$q u'\left(c_k\right) = \beta \left(1 - \rho + \rho u'\left(c_{k+1}\right)\right), \quad k = 1, \ldots, D-2, \tag{12}$$

$$q u'\left(c_{D-1}\right) \geq \beta, \tag{13}$$

where $c_k,\ k = 1, \ldots, D-1$ denotes the consumption level of a participating country that is unproductive for the k consecutive prior periods. The budget constraints (1) become:

$$B_{k-1} = c_k - \bar{l} + q B_k,\ k = 1, \ldots, D-2, \tag{14}$$

$$B_{D-2} = c_{D-1} - \bar{l} + q(1-\rho) B_{D-1} + q\rho \underline{B}, \tag{15}$$

We denote by l_{k0} the labor supply of productive countries that have been unproductive for the previous k periods and have just returned to productivity. By extension, l_{00} denotes the labor supply of productive countries that were already productive in the previous period, and l_{-10} is the labor supply of participating productive countries that were excluded from financial markets in the previous period and that have just reentered the financial markets. Recall that the beginning-of-period wealth of these countries will also be assumed to be null: $B_{-1} = 0$. The budget constraint (1) implies:

$$l_{k0} = u'^{-1}\left(1\right) - B_k + q B_0,\ k = -1, 0, \ldots, D-1, \tag{16}$$

The intuition for this simple equilibrium structure is that all productive countries choose the same consumption level $u'^{-1}\left(1\right)$ and the same asset holding B_0 due to the quasi-linearity of the utility function (and the linearity of the value function). The labor supply of productive countries l_{k0} adjusts to compensate for the difference in their beginning-of-period wealth levels.

This wealth level depends on the country's productive status in the previous period and in particular on the number k of consecutive past periods of being unproductive (with $k = 0$ corresponding to a productive status and $k = -1$ to a default status).

Remark 1 (Notation) *To lighten the notation, in the remainder of the article we consistently employ the notation introduced in this section. For a variable x, x_k is the quantity associated with a country that is unproductive for $k = 1, \dots, D-1$ periods, while x_{k0} denotes the quantity for a productive country that was unproductive for the $k = -1, 0, \dots, D-1$ previous periods. For instance, x_{20} concerns a productive country that was unproductive in the last two previous periods. The quantity associated with a productive country that was already productive in the previous period is x_{00}, while x_{-10} refers to a productive country that was excluded. When the past status has no influence for a productive country – for instance in the case of consumption level – we simply use the notation x_0.*

3.2 Default conditions and limited-heterogeneity equilibrium

Countries will default after being unproductive for exactly D consecutive periods if the following conditions hold: (i) paying back debt dominates the default option if the countries are unproductive for less than $D-1$ consecutive periods; (ii) countries unproductive for exactly D consecutive periods would be worse off paying back their debt than defaulting and paying $\underline{B}$; (iii) productive countries that have previously been unproductive for k consecutive periods choose to pay back their contracted debt while unproductive and do not default. This implies:

$$V_u^c(B_k) \geq V_u^d(\underline{B}),\ 0 < k < D, \tag{17}$$

$$V_p^c(B_k) \geq V_p^d(\underline{B}),\ 0 < k < D, \tag{18}$$

$$V_u^d(\underline{B}) \geq V_u^c(B_D), \tag{19}$$

where the utility in cases of default depends on the debt repayment $\underline{B}$. Note that the inequality (19) implies that the value $\underline{B}$ cannot be too negative. In other words, the recovery ratio $\underline{B}/B'$ has to remain sufficiently small.

We can now express the financial market clearing condition. First, we denote by n_k the population of participating countries that are unproductive for k consecutive periods for $1 \leq k \leq D$. From transition probabilities, we can state that $n_k = \rho^k(1-\alpha)n_0$ for $k = 1, \dots, D-1$, where

n_0 denotes the mass of productive countries participating in financial markets. In the interests of conciseness, the explicit expression of these population shares is shown in Appendix A. Using this notation and substituting for equilibrium population shares and prices, the financial market equilibrium (8) can be expressed as follows:

$$\sum_{k=0}^{D-2} n_k B_k + (1-\rho) n_{D-1} B_{D-1} + \rho n_{D-1} \underline{B} = 0. \tag{20}$$

To summarize, the equilibrium is thus characterized by: 1) a number of periods D before default, with the possibility that $D = \infty$, if default never occurs; 2) consumption levels $(c_k)_{k=0,\ldots,D-1}$, asset demands $(B_k)_{k=0,\ldots,D-1}$, and labor efforts $(l_{k0})_{k=-1,0,\ldots,D-1}$ satisfying Euler equations (11)–(13) and budget constraints (14)–(16), while conditions (17)–(19) hold; 3) a price for the safe asset q, enabling the market to clear and equation (20) to hold.

As we show in our quantitative exercise in Section 7, the value D is finite for standard calibrations. The economy is thus populated by a finite number of different types of countries, where each type chooses the same consumption and wealth. More precisely, $D-1$ unproductive types participating in financial markets are characterized by the number $k = 1, \ldots, D-1$ of consecutive periods for which they are unproductive. Furthermore, $D+1$ productive types participate in financial markets and are characterized by their status in the previous period, which could be either: (i) unproductive, (ii) productive, or (iii) excluded from the financial markets. As explained above, productive countries choose the same consumption and asset holding and only differ with respect to their labor effort. In addition to these $2D$ participating types, 2 additional types of country are excluded from the financial markets, and differ according to their productive status. On the whole, the economy is populated by $2D+2$ different types of countries.

We can now compute the welfare for each type of country. Consistent with our previous notation, we denote by V_k the intertemporal welfare of a country participating in financial markets and unproductive for $k = 1, \ldots, D-1$ periods, while V_{k0} denotes the intertemporal welfare of a productive country participating in financial markets, which was productive in the

$k = -1, \ldots, D-1$ previous periods. Formally, we have:

$$V_{k0} = u\left(u'^{-1}(1)\right) - l_{k0} + \beta\left(\alpha V_{00} + (1-\alpha)V_1\right), \; k = -1, \ldots, D-1, \tag{21}$$

$$V_k = u\left(c_k\right) - \bar{l} + \beta\left((1-\rho)V_{k0} + \rho V_{k+1}\right), \; k = 1, \ldots, D-1. \tag{22}$$

Using expressions (21) and (22), participation conditions (17) and (19) become $V_k \geq V_u^d(\underline{B})$ and $V_{k0} \geq V_p^d(\underline{B})$ for $k = 1, \ldots, D-1$.

The next proposition further characterizes the equilibrium structure.

Proposition 1 (Limited-heterogeneity equilibrium characterization) *The price q verifies $q \geq \beta$ and we have* either*:*

(a) $D = \infty$, $q > \beta$ *and* $\lim_{k\to\infty} B_k = -\frac{\bar{l}}{1-q}$; or

(b) $D < \infty$ *and* $V_p^c(B_{D-1}) = V_p^d(\underline{B})$.

Proposition 1 states that two types of equilibrium can exist. The first (a) is an equilibrium in which default never occurs. The second (b) is an equilibrium in which the borrowing limit B_{D-1} for unproductive countries is determined by their default incentive when becoming productive in the next period and repaying $\underline{B}$. In this equilibrium, unproductive countries borrow increasing amounts while remaining unproductive, until they reach an amount that would lead them to default in the following period if they became productive. A productive country that starts the period with a large amount of debt would have to provide a significant labor effort to repay its debt and it may thus be better off defaulting.

Despite characterizing a possible equilibrium structure, Proposition 1 is silent about the equilibrium uniqueness. Sections 4 and 5 present and discuss equilibrium multiplicity.

Finally, we will compare the multiple equilibria using a utilitarian welfare criterion. Other criteria could be used, but this one seems to be the most natural in heterogeneous-agent economies, for reasons already discussed in Aiyagari (1994), and it is thus widely used in the literature. Using the share of different types of countries, the expression of aggregate welfare W^a is simply:

$$W^a = \sum_{k=-1}^{D-1} n_{k0}V_{k0} + \sum_{k=1}^{D-1} n_k V_k + n_p^d V_p^d(0) + n_{u,1}^d V_u^d(\underline{B}) + \left(n_u^d - n_{u,1}^d\right) V_u^d(0), \tag{23}$$

where n_s^d $(s = p, u)$ is the number of countries in state s that have defaulted and are in autarky, and $n_{u,1}^d$ is the number of unproductive countries that have just defaulted, and that repay $\underline{B}$.

Aggregate welfare is the sum of the welfare of countries participating in the financial markets, whether they are productive or unproductive, and the welfare of countries excluded from the financial markets, again whether they are productive or unproductive.

4 Identifying mechanisms in a simpler set-up

This section provides introductory insights on two core aspects of the model: (i) the existence of multiple equilibria (Section 4.1), and (ii) the benefits of liquidity provision by an International Financial Institution (Sections 4.2 and 4.3). For the first point, we provide a detailed example of an economy featuring two distinct equilibria, differing with respect to the amount of liquidity and the number of consecutive unproductive periods before the default. Regarding the second point, we show that liquidity provision improves the aggregate welfare.

For this identification section, the model is simplified along two dimensions.

1. First, the amount recovered in case of default is set to zero, $\underline{B} = 0$. This assumption is innocuous, as positive recovery rates are mainly introduced for quantitative purposes.

2. Second, the probability to reenter the financial market after default is set to 0: $\theta = 0$. To maintain a constant size of the participating population, it is assumed that when a country defaults, a new country "enters" into the economy with zero wealth. The initial status of this new country is productive with a probability n^p and not productive $1 - n^p$. As a consequence, both the size and the structure of the participating population are constant. Furthermore, the welfare after default does not depend on prices or on the risk-sharing arrangements of participating countries, since defaulting countries are excluded forever. This removes one fixed-point in the default decision, which simplifies the algebra. We check in Section 4.4 that allowing countries to reenter financial markets (i.e., allowing for $\theta > 0$) preserves model properties.

We choose model parameters such that we can focus on the simplest equilibrium cases $D = 2$, and $D = 3$, where countries default after either 2 or 3 periods in the unproductive status. We start with defining the volume of demand and supply for savings as a function of the asset price

q assuming that $D = 2$. These functions are denoted as Δ_2 and Σ_2 respectively.

$$\begin{cases} \Sigma_2(q) & = n_0 B_0^+(q) + (1-\rho)(1-\alpha) n_0 B_1^+(q), \\ \Delta_2(q) & = -\left(n_0 B_0^-(q) + (1-\rho)(1-\alpha) n_0 B_1^-(q)\right), \end{cases} \tag{24}$$

where x^+ and x^- denote $\max(x,0)$ and $-\min(x,0)$, respectively, such that $x = x^+ - x^-$. In equation (24), we use this notation to split net savings into actual gross savings and borrowings.

We similarly define supply and demand functions when $D = 3$.

$$\begin{cases} \Sigma_3(q) & = n_0 B_0^+(q) + (1-\alpha) n_0 B_1^+(q) + \rho(1-\rho)(1-\alpha) n_0 B_2^+(q), \\ \Delta_3(q) & = n_0 B_0^-(q) + (1-\alpha) n_0 B_1^-(q) + \rho(1-\rho)(1-\alpha) n_0 B_2^-(q). \end{cases} \tag{25}$$

Finally, the total demand and supply functions, denoted Σ and Δ respectively, are:

$$\Delta(q) = \Delta_2(q)\,\mathbf{1}_{D=2} + \Delta_3(q)\,\mathbf{1}_{D=3}, \text{ and } \Sigma(q) = \Sigma_2(q)\,\mathbf{1}_{D=2} + \Sigma_3(q)\,\mathbf{1}_{D=3},$$

where for instance, $\mathbf{1}_{D=3} = 1$ if the optimal number of default periods is 3 and 0 otherwise.

The calibration to generate an equilibrium with either $D = 2$ or $D = 3$ involves $u(c) = \log c$, $\beta = 0.7$, $\bar{l} = 0.3$, $\alpha = \rho = 0.8$, $\varphi = 0.9$.[7] A more empirically relevant calibration is provided in Section 7.

4.1 Equilibrium multiplicity

We plot in Figure 1 the demand and supply functions for different values of q, varying in a range $[0.87, 0.95]$, which is the relevant range for equilibrium multiplicity. To do this, for each price value q, we solve the countries' programs and compute the optimal D. The financial market clearing condition, corresponding to equation (20), is $\Sigma(q) = \Delta(q)$ and holds when supply and demand curves intersect.

We can draw several lessons from Figure 1. First, we check that for any value of q in this range, agents default after either $D = 2$ or $D = 3$ periods in the unproductive state. As a consequence, in partial equilibrium (fixed q and no market clearing) the equilibrium is unique, which is consistent with the findings of Auclert and Ronglie (2016) in a related environment without labor supply.

[7] It could be possible to provide analytical expressions for the supply and demand functions when $D = 2$ or $D = 3$. However, the simple calibration is sufficient to convey all major economic insights.

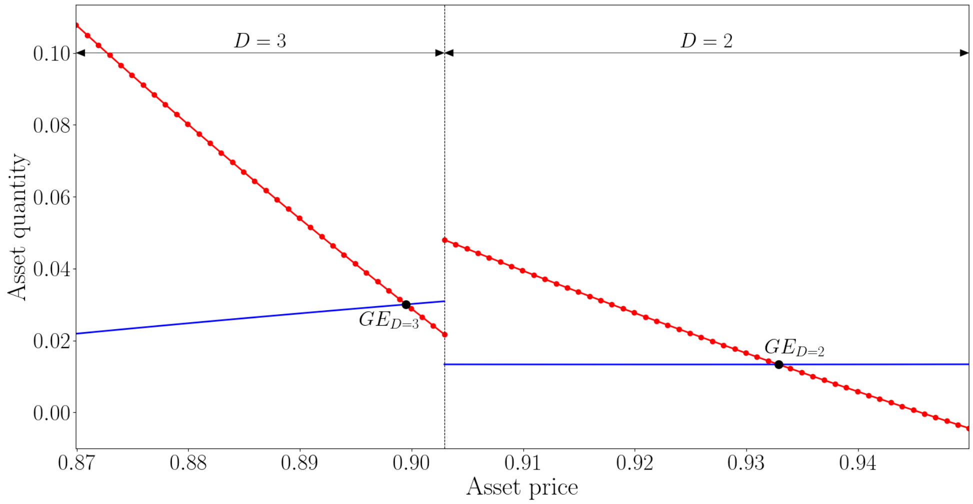

Figure 1: Supply (red dotted line) and demand (blue solid line) for assets as a function of the price q. See the text for parameter values.

Second, the two zones corresponding to the two values of D can be clearly seen in Figure 1. The change in the optimal D generates a discontinuity in demand-supply functions at the price q_d, approximately equal to 0.902. When $q > q_d$, then $D = 2$ and when $q < q_d$, $D = 3$. When q *decreases* from $q_d + \varepsilon$ to $q_d - \varepsilon$ (with $\varepsilon > 0$ small), D increases from 2 to 3. The intensive effect of a lower liquidity price enables each country to better self-insure itself, which triggers at that particular price a change in the default decision: countries find it optimal to default after $D = 3$, instead of $D = 2$ periods. This has an extensive effect on liquidity demand, since because of a higher D, a larger number of countries are borrowing: the demand for liquidity (blue line) jumps upwards. This also has an extensive effect on liquidity supply since countries consider now the possibility to borrow when $D = 2$ (instead of defaulting) as a sudden relaxation of their future borrowing limits. This diminishes countries' self-insurance needs. This extensive effect dominates the intensive one (related to the lower liquidity price), and the supply of liquidity therefore jumps downwards (red line) when D goes from $D = 2$ to $D = 3$. As a direct implication, both the demand and supply curves for liquidity are not monotonic due to the endogenous default choice.

Finally, supply and demand curves intersect in each of the two zones $D = 2$ and $D = 3$. There are therefore two general equilibria: one for $D = 2$ with a high price and a low inside liquidity, another for $D = 3$ with a low price and a high inside liquidity. The key mechanism for multiple equilibria is thus general equilibrium and the endogeneity of asset prices. When the safe asset is scarce, its price q is high and the risk-free interest rate r is low. Self-insurance is expensive as the return on savings is low and countries have little incentive to save in good times for self-insurance motives. Countries then default earlier and the demand for liquidity is low. Both supply and demand are therefore low when the price of liquidity is high (right part of Figure 1). Conversely, when there is abundant liquidity, the welfare cost of self-insurance is low, the interest rate is high (q is low), and countries save to self-insure more and default less often, thus borrowing more. The quantity D is high in equilibrium (left part of Figure 1) at a low liquidity price.

4.2 The effect of outside liquidity: Introducing an International Financial Institution (IFI)

We now analyze the consequence of exogenously increasing outside liquidity in the previous economy. An international financial institution (IFI) is assumed to issue a quantity of debt $B^{IFI} < 0$. This debt is riskless and promises to repay one unit of goods in the next period. As a consequence, the IFI must find resources to repay $T = B^{IFI}(q-1) > 0$ in each period. For the sake of simplicity, we assume that the IFI has an exogenous source of income equal to T. This assumption is relaxed in Section 4.3. The clearing of the financial market can be expressed as: $\Sigma(q) = \Delta(q) + (-B^{IFI})$, where the exogenous debt increases the demand for liquidity. Figure 2 plots the new supply and demand curves for $B^{IFI} = -0.06$ with the previous calibration.

We can see in Figure 2 that B^{IFI} shifts upward the demand for liquidity. There are two consequences. First, the $D = 2$ equilibrium disappears and only the high risk-sharing equilibrium $D = 3$ exists. It is noteworthy that the non-existence of the $D = 2$ equilibrium depends on the size of the IFI: for low outside liquidity, the equilibrium still exists, while it does not as soon as the outside liquidity goes beyond a threshold (as in our calibration). Second, the price of liquidity decreases and the total liquidity increases compared to the economy of Section 4.1. Risk-sharing among countries improves and countries are better off. The intensity of this effect is stronger when the size of the IFI gets bigger. Larger liquidity supply thus comes with better risk-sharing and larger welfare.

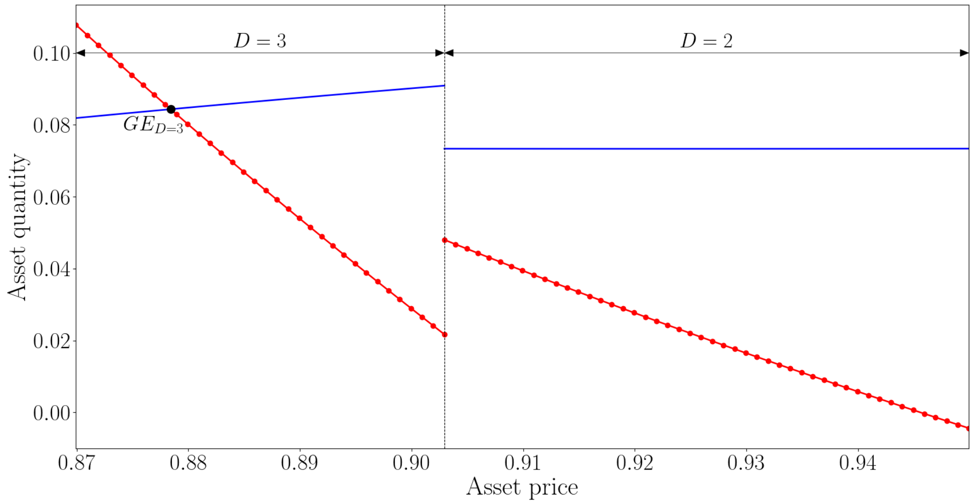

Figure 2: Supply (red dotted line) and demand (blue solid line) for assets as a function of the price q, with exogenous international liquidity. See the text for parameter values.

4.3 Financing outside liquidity

We now introduce a simple world tax structure to finance outside liquidity, to check that the previous results are robust in this case. We assume that the IFI can tax all countries participating in the financial market, i.e., all countries that have not defaulted. Furthermore, the tax rate, denoted τ, must be constant for all countries and cannot depend on their productive status. This simple tax structure relies on the assumption that the IFI can force countries to pay taxes.

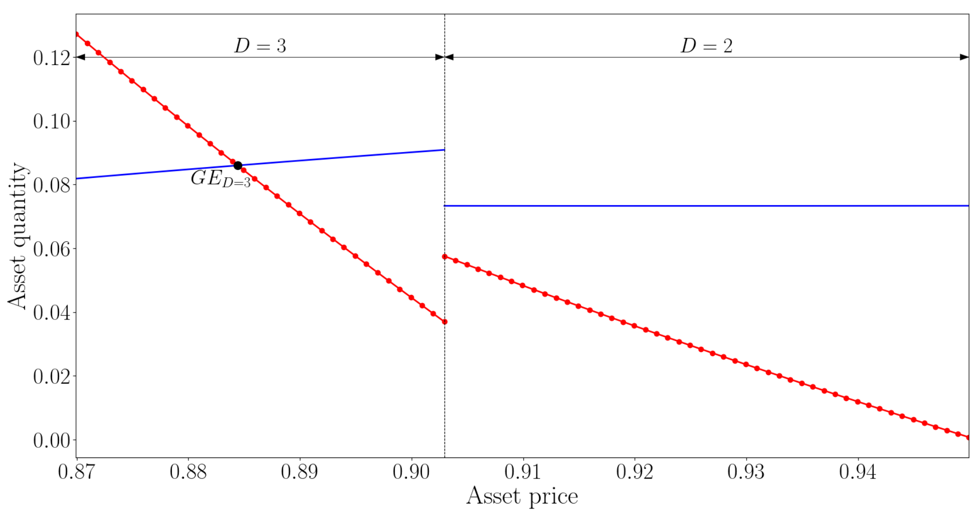

Figure 3: Supply (red dotted line) and demand (blue solid line) for assets as a function of the price q, with financed international liquidity. See the text for parameter values.

Figure 3 plots the new supply and demand curves, with the same calibration as in Section 4.2, except that outside liquidity is financed. Compared to the non-financed case of Section 4.2, the equilibrium price of debt increases, roughly to $q = 0.885$ from $q = 0.877$ in the unfinanced case. Similarly, the total liquidity diminishes in presence of the tax (as countries are poorer and save less), which diminishes risk-sharing. It is not surprising that financing the liquidity comes with a cost. However, compared to the absence of IFI in Section 4.1, financed outside liquidity still enhances welfare by increasing risk-sharing: the asset price goes down and the total liquidity in the economy increases.

This experiment also confirms the non-Ricardian nature of the economy with endogenous default, as the supply and demand curves in Figure 3 differ from those in Figure 1. Indeed, unlike in the seminal paper of Barro (1974) on debt neutrality, our economy is non-Ricardian for two reasons. First, in the period right before default occurs, countries face a binding credit constraint, as already noted by Arellano (2008). Second, as countries can default in equilibrium, a country in autarky after default will not pay any tax, which breaks debt neutrality.

4.4 Allowing for reentry after default and for recovery

We now remove the assumption that $\theta = 0$, and consider a constant total population of size 1. Productive countries in autarky can reenter the economy with a probability $\theta > 0$. This has two effects. First, it increases the value of autarky in each state, and makes it dependent on the risk-sharing arrangement of participating countries. Second, it affects the size of the population in each state. This last effect is not crucial for our equilibrium structure. Indeed, supply and demand functions are scaled by the number of productive participating countries (denoted n_0), as can be seen from equations (24) and (25). In a similar vein, the introduction of an exogenous recovery rate $\underline{B} < 0$ reduces the gain of default and thus diminishes the incentives to default.

We can check that the previous analysis of Sections 4.1–4.3 remains valid when reentry or a positive recovery rate are taken into account with a plausible calibration. As an illustration, we plot in Figure 4 the financial market equilibrium without IFI in the case of a reentry probability $\theta = 2\%$. This is the parallel of Figure 1. One can observe that there are still two equilibria: one for $D = 2$ with a high price and a low inside liquidity and another one for $D = 3$ with a low price and a high inside liquidity (and a higher aggregate welfare). The existence of multiple equilibria is thus a quantitative outcome, and the increase in welfare implied by a larger liquidity supply is a robust finding.

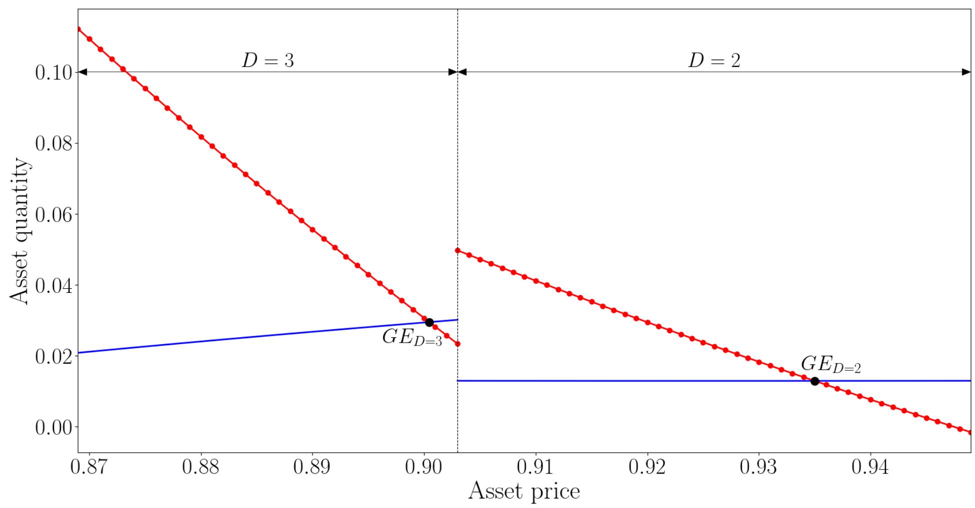

Figure 4: Supply (red dotted line) and demand (blue solid line) for assets as a function of the price q, with positive probability to reenter after default. See the text for parameter values.

5 Welfare analysis and constrained-efficiency in the general case

We now provide results in the general case (i.e. $\underline{B}, \theta > 0$), concerning the ranking of equilibria (Section 5.1), and the structure of constrained-efficient equilibria (Section 5.2).

5.1 Ranking of multiple equilibria

As shown above in a special case, the market economy can generate multiple equilibria. This robust finding is a general consequence of general equilibrium and of the endogeneity of asset prices. The identification of this mechanism for equilibrium multiplicity in models with incomplete markets and default is new in the literature, to the best of our knowledge. Other mechanisms can be found in the literature as examined in the literature review above. To summarize, they either arise because of the refinancing risk (Calvo 1988, Cole and Kehoe 2000, Lorenzoni and Werning 2013, or more recently He, Krishnamurty, and Milbradt 2019), or because of some strategic complementarity between a small number of countries and risk adverse lenders (Arellano, Bai, and Lizarazo 2018). We instead focus here on general equilibrium effects.

We prove that these multiple equilibria can be ranked according to their aggregate welfare.

Proposition 2 (Ranking of multiple equilibria) *The higher the number D of consecutive periods before default, the higher the aggregate welfare will be. As a consequence, the competitive equilibrium that maximizes aggregate welfare is that with the largest admissible D.*

The proof can be found in Appendix C. Intuitively and as in Figures 1 and 4, the economy with a lower default amount (and a higher D) is characterized by a higher quantity of liquidity and therefore more risk-sharing, which is welfare-improving.

5.2 Constrained-efficiency in the market economy

We now show that the market equilibrium with the largest D is not constrained-efficient. The liquidity remains indeed insufficient for constrained-efficiency, because of market incompleteness.[8] To show this formally, we follow the methodology of Davila, Hong, Krusell, and Rios-Rull (2012), who define a relevant notion of constrained-efficiency in an incomplete insurance-market environment without default. We extend this to an economy with default, imposing an additional participation constraint on the quasi-planner.[9] The idea is to solve for the allocation, when a quasi-planner internalizes the effect of liquidity choices on interest rates (agents are no longer price-takers). This analysis not only compares steady-state welfare levels, but also considers the cost of transition to different steady-states.[10]

More formally, the quasi-planner can choose how much each country can save, consume, and work, but it cannot transfer resources across countries (country-specific budget constraints must hold) and cannot prevent countries from defaulting (countries' participation constraints must also hold). The quasi-planner also internalizes the financial market clearing condition. The optimal allocation of the quasi-planner defines the constrained-efficient allocation when the pecuniary externality is internalized.

The constrained-efficient allocation of the quasi-planner is summarized in the next proposition. To save some space, the formal presentation of the economy can be found in Appendix D, and the proof of the proposition is provided in Appendix E.

[8]Davila, Hong, Krusell, and Rios-Rull (2012) show that the market equilibrium is not constrained-efficient in an incomplete-market model with borrowing constraint, and no default. One can see our result as an extension to an economy with default. Constrained-inefficiency should thus be seen as the outcome of market incompleteness in various environments (with or without default).

[9]The term quasi-planner refers to a planner maximizing social welfare with country-specific budget constraints as in Veracierto (2008) in a different environment, for instance.

[10]Comparing steady-state welfare is indeed not a consistent way to rank allocations, as it does not properly consider consumption smoothing. See Le Grand and Ragot (2017) for a discussion and additional references.

Proposition 3 (Constrained-efficient allocation) *The constrained-efficient allocation is either autarky or is characterized by a default horizon* $\tilde{D}$ *and allocations* $\tilde{c}_k = u'^{-1}(1)$ *for* $k = 0, \ldots, \tilde{D} - 1$.

There are two possible constrained-efficient allocations. First, one cannot exclude the possibility that autarky might be the only equilibrium, following the results of Bulow and Rogoff (1989). This is the case if the cost of default is low. Second, when the cost of default is high enough, default occurs in equilibrium (which is the case in the numerical examples of Section 4 above or of Section 7 below). In this case, the quasi-planner chooses that default occurs after $\tilde{D}$ periods in unproductive status. The optimal outcome is then to maximize the safe asset quantity and to implement full risk-sharing among participating countries. This increases the welfare of participating countries, and therefore reduces the default incentives. As a direct consequence, the constrained-efficient allocation in general differs from the market allocation characterized in Section 3.1. To see this, observe that $\tilde{c}_k = u'^{-1}(1)$ in the market economy implies $q = \beta$. However, in this case, there is no reason for the saving decisions to satisfy the financial market clearing condition. Since the quasi-planner internalizes the financial market clearing condition, it distorts individual saving decisions to reach full risk-sharing, while allowing financial markets to clear.

6 Increasing risk-sharing: Designing an international Financial Institution (IFI)

As the market equilibrium is not constrained-efficient, we now present two institutional arrangements to increase risk-sharing through outside liquidity provision. The first arrangement involves an IFI that issues world liquidity and implements full risk-sharing among its members, as in the constrained-efficient allocation. As the commitment device of the fund required to implement this allocation may be unrealistic, we provide a second institutional arrangement, based on a weaker commitment device. The optimal amount of liquidity provided by both institutions is quantified in the numerical exercise of Section 7.

We now present the structure of the IFI, which is common to the two institutional arrangements studied below. The environment is the same as in Section 2 (we keep the same notation).

The IFI issues one-period, risk-free debt financed by the contributions of member states. As in Section 4, the IFI transfers are forbidden from being contingent on country's productive status. Otherwise the IFI could implement the first best allocation by completing the market. The IFI has therefore no comparative advantage compared to the financial markets.[11] Compared to the simple case in Section 4, we make here the following assumptions regarding the IFI.

1. The IFI can issue debt and is financed solely by the contributions of member countries.

2. Countries can freely choose to belong to the IFI and can always choose to opt out.

3. Contributing countries agree not to transact with countries not contributing to the IFI.

First, the IFI budget is financed only by the contributions of countries. Second, countries can decide whether to participate in the IFI or not. The participation decision is sovereign. Third, contributing countries agree, when joining the IFI, not to transact with non-contributing countries. Otherwise, countries would have no incentive to pay their contribution to the IFI, as they could benefit from outside liquidity (and pay the world interest rate on any safe asset through the absence of arbitrage) without paying any contributory costs.

We denote by F' the new debt issued by the IFI at price q in a given period, while F is the amount the IFI repays. Note that the IFI pays the world risk-free interest rate due to the absence of arbitrage. The measure of countries participating in the IFI is denoted by m_p. Countries participating in the IFI pay τ. Using this notation, the IFI budget constraint is:

$$F = qF' + m_p \tau. \tag{26}$$

The world financial market clearing condition can now be expressed as:

$$\sum_{k=0}^{D-2} n_k B_k + (1-\rho) n_{D-1} B_{D-1} + \rho n_{D-1} \underline{B} = F'. \tag{27}$$

Finally, the budget constraints of countries are the same as (14)–(16), except that countries additionally contribute to the IFI by an amount τ.

[11] One could argue that the ability to lend to countries in crisis against some credible commitment from these countries (a program) is a comparative advantage for certain international institutions, such as the IMF. This comparative lending advantage helps to complete the market. We do not consider this role here, but instead focus solely on the supply of liquidity.

6.1 Implementing full risk-sharing

We introduce an additional assumption for the IFI to implement full risk-sharing among its members. We further assume that:

4. Contributing countries can commit to hold a maximum amount of liquidity.

This fourth assumption is necessary for the equilibrium with full risk-sharing to exist.

As shown in Proposition 4 below, full-risk sharing can be implemented for all countries participating in the IFI. Indeed, to implement full risk-sharing, the IFI chooses the volume of its debt such that the equilibrium price of liquidity is $q = \beta$. In this case, the Euler equations of participating countries imply that their marginal utility of consumption is 1. At this price, however, all productive countries would decide to work a lot to buy a huge amount of liquidity to self-insure against fluctuating income. Indeed, there is no opportunity cost to save for consumption smoothing and expected uninsurable shocks always generate an additional incentive to save. This outcome is the standard result of Bewley (1983) about saving decisions with precautionary motives. But this deviation would prevent the existence of the equilibrium, as the IFI would have to issue a huge amount of liquidity. To prevent this deviation, the IFI must be able to limit the savings ofcountries participating in the risk-sharing arrangement. With this limited amount of liquidity, one can check that the country is still willing to join the risk-sharing arrangement. This explains why Assumption 4 is needed for equilibrium existence.

Proposition 4 (Equilibria in family-head and decentralized economies) *We consider the introduction of an IFI, where assumptions 1 to 4 hold. For finite D, there exists a value of F such that the marginal utility of consumption of all participating countries is* 1 *among countries participating in the IFI, and risk-sharing is thus perfect.*

The proof can be found in Appendix F, where we provide existence conditions. The equilibrium quantity F is determined to guarantee that the asset price is $q = \beta$. In this equilibrium, productive countries save to participate in the risk-sharing arrangement, and unproductive countries dis-save until countries optimally choose to default. Even in presence of Assumption 4 above, the proposition remains silent regarding existence. Indeed, existence is not guaranteed in the general case, as the contribution to the IFI needed to balance the IFI budget constraint may be too high for countries to be willing to join the risk-sharing arrangement, even when in autarky. Instead, we provide below a numerical example to show existence and the properties of such an IFI in a quantitatively relevant environment. The IFI can be seen as a pure financial actor targeting the proper interest rate using the size of its balance sheet in this non-Ricardian environment.

6.2 The maximum provision of outside liquidity

We now relax Assumption 4 of the previous Section and only preserve Assumptions 1 to 3. In this case, the IFI cannot implement full-risk sharing among its members. Nevertheless, the IFI can improve risk-sharing by issuing liquidity and decreasing the world price of liquidity. This allocation features $q > \beta$, which ensures equilibrium existence. We define the maximum provision of outside liquidity as the highest value of liquidity F consistent with the equilibrium existence. The binding equilibrium existence condition is that the contribution of countries must be low enough for countries to accept to join the risk-sharing arrangement when they get out of autarky (at a rate θ).[12] We provide existence conditions in Appendix F. We use these conditions to determine the maximum amount of outside liquidity in the quantitative exercise below. We show below that the size of the IFI is much smaller in this second case. However, since this allocation does not require a strong commitment device for the IFI – as was the case of Assumption 4 –, this second allocation may thus appear to be more realistic than the one implementing full risk-sharing in Section 6.1.

6.3 Comparison with Special Drawing Rights

This IFI is obviously reminiscent of the IMF issuing Special Drawing Rights, which are international stores of value. SDRs are reserve assets and have been issued by the IMF since 1970, with the explicit goal of reducing the world liquidity shortage (see Williamson 2009 for a summary of the history of the introduction of SDRs). A first difference between the IFI introduced in this section and SDRs is that the interest rate on assets issued by the IFI is a yearly interest rate comparable to the one on sovereign debts, whereas the remuneration of SDRs is an average of short-term (3 months) interest rates on a basket of currencies. This remuneration is thus closer

[12] The construction of our equilibrium ensures that participating countries never choose to leave the IFI risk-sharing arrangement.

to the one of monetary markets.[13] A second difference between the IFI debt and the SDRs is the outstanding volume of debt, as illustrated in the numerical example of Section 7 below.

7 A numerical example

To provide further insights, we now perform a quantitative exercise to investigate equilibrium multiplicity and to quantify the size of the IFI required to maximize risk-sharing. To calibrate the model we use the literature on both sovereign default and on disaster events. The period is one year. The model has eight parameters, for which eight targets or values are provided. The calibration and target values are summarized in Table 1.

Parameters	Values	Target and references
Discount factor	$\beta = 0.96$	Interest rate of 4%
Utility function	$\sigma = 2$	Arellano (2008)
Persistence of good state	$\alpha = 96.5\%$	3.5% disaster probability (Barro and Ursua 2012)
Persistence of disaster	$\rho = 87\%$	Average duration of disaster of 7.7 years (Barro and Ursua 2012)
Labor in disaster state	$\bar{l} = 0.985$	10% cumulative GDP fall on default (Tomz and Wright 2013)
Prob. of reentering	$\theta = 15\%$	7 years of exclusion after default (Tomz and Wright 2013)
Default cost	$\varphi = 0.9965$	Default after 8 years in unproductive state (Barro and Ursua 2012)
Debt recovery	$\underline{B} = -0.03$	Haircut of 28% (Tomz and Wright 2013)

Table 1: Calibration

The curvature of the utility function is set to $\sigma = 2$, as in Arellano (2008), and the discount factor is set to $\beta = 0.96$ to generate an annual world interest rate of around 4%. We set the probability of remaining productive to 96.5% such that the probability of moving from a

[13]The weekly interest rates on SDRs are provided at `http://www.imf.org/external/np/fin/data/sdr_ir.aspx`, together with the explicit interest rate formula. The current level of outstanding IMF SDRs, which amounted to less than 0.3% of world GDP in 2016.

productive to unproductive state is 3.5%. This is equal to the annual probability of entering into a disaster state, as found by Barro and Ursua (2012). The probability of remaining unproductive is set to 0.87% such that the average number of periods spent in an unproductive state amounts to 8 years. This value is consistent with the average duration of a disaster state provided by Barro and Ursua (2012). The production when unproductive is set to $\bar{l} = 0.985$, which implies a cumulative output loss before default of around 10%. This is consistent with Tomz and Wright (2013), who report a cumulative loss of 8% when default occurs. The probability of reentering the economy is set to $\theta = 15\%$, which generates an average length of financial market exclusion of 7 years, which is the length reported by Tomz and Wright (2013). The cost of default is set to 0.35% of output ($\varphi = 0.9965$). This value is chosen to trigger default after 8 consecutive periods in the unproductive state, which is the average length of a disaster. This value implies that 13% of all countries are in default, which is not far from the empirical value of 19% found by Tomz and Wright (2013). Finally, $\underline{B}$ is -0.03 to generate a realistic recovery rate close to 30% (Tomz and Wright 2013). A consequence of this calibration is that the unconditional probability of default amounts to 1%. This value is slightly below empirical estimates, which vary between 1.8% and 2.2%.

7.1 Market equilibria

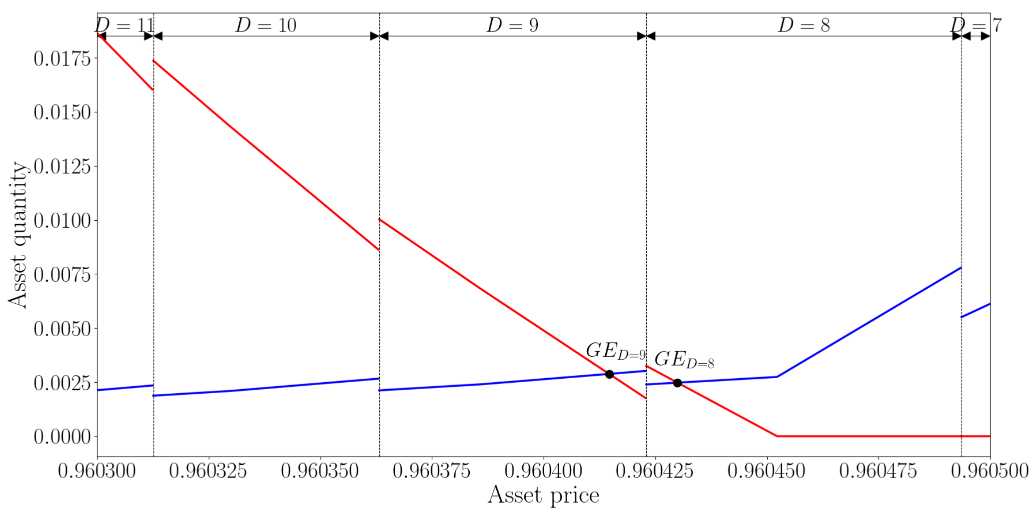

Figure 5: Supply (red line) and demand (blue solid line) for assets as a function of the price q in the calibrated economy.

We start with plotting in Figure 5 the supply and demand of liquidity for various prices of debt q, as we did in Figure 1 of Section 4. For each price, we optimize over the default horizon D and all savings decisions. As for Figure 1, the discontinuities of the supply and demand curves correspond to changes in the optimal default horizon D. When the price of debt is higher than the threshold of approximately 0.96045, the supply curve is flat at 0, since the interest rate becomes low enough for all countries to borrow.

As in Section 4, an equilibrium corresponds to an asset price, for which the financial market clears: supply equals demand. We can observe that the calibrated model generates two equilibria, corresponding to the two intersection points in Figure 5. The two equilibria correspond to either $D = 8$ or $D = 9$. The equilibrium with $D = 9$ is associated with a higher inside liquidity (and thus a better risk-sharing) and a lower price of debt (and thus a higher interest rate), even though the difference between the two interest rates is small, around 0.5 basis point. In conclusion, both equilibria therefore feature limited risk-sharing, but the one with a higher D features a higher inside liquidity and thus a better risk-sharing. This is consistent with the results of Proposition 2 about the ranking of multiple equilibria.

7.2 The size of the IFI implementing full risk-sharing

We now compute the optimal size of the IFI that implements full risk-sharing among its members, with the assumption of Section 6.1. Iterating over the IFI size F, we find that the welfare-maximizing value for the default horizon is $D^{opt} = 18$: countries default after being unproductive for 18 periods. The world interest rate is 4.17%. All countries participating in financial markets, either productive or not, enjoy the same consumption, equal to 1. There is thus full risk-sharing for countries participating in the financial markets. Proposition 4 is thus verified in our empirical exercise.

This value of D is attained for a size of the IFI equal to 29.4% of world GDP. The average contribution of a country to the IFI is equal to approximately $\tau = 1.22\%$ of its own GDP. In this economy, the fraction of countries defaulting is 0.22%, and only 3.8% of the countries are excluded from financial markets. Compared to the equilibrium with $D = 9$, we find that the welfare gain of introducing the IFI represents for countries an average increase in consumption of 0.95% compared to the consumption in the market economy.[14] The welfare gain provided by

[14]We compute the increase in consumption in the market economy for all countries to have the same aggregate

	Market	Full risk-sharing	Max. liquidity
D	9	18	17
Interest rate (%)	4.12	4.17	4.15
% countries in the default state	12.7	3.8	4.3
ΔWelfare (consum. eq., %)	0	0.95	0.30
Avg country contribution (% GDP)	0	1.22	0.38
Size IFI (% of World GDP)	0	29	9

Table 2: Comparison of three economies: market equilibrium, IFI implementing full risk-sharing and IFI choosing the maximum liquidity. The line ΔWelfare reports the difference in welfare in terms of consumption equivalents compared to the market economy.

the IFI introduction comes from two margins, an intensive and an extensive one. First, countries are better insured in the IFI economy when they participate in financial markets, which is the intensive margin. Second, fewer countries default in the IFI economy as D is higher, which is the extensive margin. We report the result in the second column of Table 2, entitled "Full risk-sharing".

7.3 Optimal provision of liquidity

We now compute the maximum provision of liquidity by the IFI, when it cannot control countries' savings. Full-risk sharing can therefore not be implemented. Results can be found in the third column of Table 2, entitled "Max. liquidity". We find that the maximum liquidity amounts to 9% of world GDP, and that the number of consecutive periods in the unproductive state before default is $D = 17$. The world interest rate is lower (i.e., the price of liquidity is higher) than in the full risk-sharing case and equal to 4.15%. In this allocation, the fraction of defaulting countries is 0.25% and the share of countries in the default state is 4.33%. The increase in welfare compared to the market economy is equal to 0.3% in consumption equivalents. Although the share of countries participating in the risk-sharing arrangement is close to the one with full risk-sharing (17 and 18 respectively), the higher liquidity price in this economy hurts self-insurance and consumption smoothing. In consequence, liquidity provision generates a smaller increase in

welfare as the one of the IFI allocation. Doing so, we keep constant the policy rules in the market economy (and thus the default decision). This is the standard computation of consumption equivalent to compare allocations in heterogeneous-agents models.

welfare compared to the full risk-sharing case.

7.4 Considering market power

The analysis above has focused on the case where every country was a price-taker and had no market power. We can check that our qualitative results are robust to the introduction of a country having some market power in the world financial markets – such a country being often called the Hegemon. Indeed, some authors argue that this may be a relevant representation of the economy (see Eichengreen 2011, or Farhi and Maggiori 2018). We check in Appendix G that our main result, stating that an IFI improves welfare, is still valid in this environment.

8 Conclusion

We provide a tractable model where sovereign default can be studied in general equilibrium. This model allows the derivation of positive and normative properties of the equilibrium structure implied by the endogenous quantity of the world safe asset. The model generates equilibrium multiplicity, which results from an endogenous world interest rate. In addition, all market equilibria exhibit insufficient risk-sharing compared to a constrained-efficient equilibrium. The additional result of the paper is that welfare is increased by the introduction of an IFI that issues safe assets based on the voluntary contributions of member states. For realistic assumption about the IFI design, the provision of liquidity by the IFI is around 9% of world GDP. The IFI size is much larger than the outstanding SDRs issued by the IMF. The qualitative results are robust to the case where a country is large enough to wield some market power on the world financial markets. The tractability of the framework allows for different extensions. For instance, one could introduce different debt maturities to investigate the properties of the equilibrium portfolio. We leave this for future research.

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Appendix

A Population shares

Using the model setup presented in Section 2.1 and the equilibrium presented in Section 3.1, we deduce the following population shares:

$$n_0 = \frac{\theta}{\theta + (1-\theta)(1-\alpha)\rho^{D-1}} \frac{1-\rho}{2-\rho-\alpha}, \tag{28}$$

$$n_k = \rho^{k-1}(1-\alpha)n_0 \text{ for } k = 1, \ldots, D-1, \tag{29}$$

$$n_k = 0 \text{ for } k \geq D. \tag{30}$$

We denote by n_u^d and n_p^d the mass of unproductive and productive agents who have defaulted and are excluded from financial markets and by $n_{u,1}^d$ the mass of countries that have just defaulted. We have the following expressions:

$$n_p^d = \frac{(1-\theta)(1-\alpha)\rho^{D-1}}{\theta + (1-\theta)(1-\alpha)\rho^{D-1}} \frac{1-\rho}{2-\rho-\alpha}, \tag{31}$$

$$n_{u,1}^d = \rho^D(1-\alpha)n_0, \tag{32}$$

$$n_u^d = \frac{(1-(1-\theta)\alpha)\,\rho^{D-1}}{\theta + (1-\theta)(1-\alpha)\rho^{D-1}} \frac{1-\alpha}{2-\rho-\alpha}. \tag{33}$$

With our timing convention, the number of excluded countries re-entering financial markets after having being excluded is then $n_{-10} = \theta\left(\alpha n_p^d + (1-\rho)\, n_u^d\right)$ in each period. It is then straightforward to deduce that the overall population (excluded and participating agents) of productive countries amounts to $\frac{1-\rho}{2-\alpha-\rho}$ and that of unproductive countries to $\frac{1-\alpha}{2-\alpha-\rho}$. Note that these populations are independent of the quantity D.

B Proof of Proposition 1

The proof can be split into three steps.

B.1 Proof that: $q \geq \beta$.

Let us assume $q < \beta$. First, if $D < \infty$, equation (13), implies $u'(c_{D-1}) \geq \beta/q > 1$ and $u'(c_k) > 1$ for $k = 1, \ldots, D-1$ by induction from equation (12). Equation (11) then yields $q > \beta$, which is a contradiction. Second, if $D = \infty$, we first show from (11) and (12) that $1 > u'(c_k)$ for all k. Equation (12) also implies $\rho(u'(c_{k+1}) - u'(c_k)) < (1-\rho)(u'(c_k) - 1) < 0$ and $(u'(c_{k+1}))_k$ is decreasing and positive. It thus converges to u'_∞, with $u'_\infty\left(1 - \frac{\beta}{q}\right) = \frac{\beta}{q}(1-\rho)(u'_\infty - 1)$ using (12). Since $q < \beta$, we must have $u'_\infty > 1$, which contradicts $1 > u'(c_k)$ for all k.

B.2 Case $D = \infty$

Assume that $q = \beta$. Using the Euler conditions (11) and (12), we recursively show that $u'(c_k) = 1$ for all k. The budget constraint then implies $u'^{-1}(1) + \beta B_{k+1} = \bar{l} + B_k$. It is then easy to show that $\lim_{k\to\infty} B_k = -\infty$ (any other case being impossible due to the financial market clearing condition). This is a contradiction with equation (9), as there is a threshold for debt in any equilibrium under consideration. In consequence $q > \beta$. We thus obtain from Euler equations (11)–(13):

$$1 - \alpha < (1-\alpha)u'(c_1)$$
$$u'(c_k) - 1 < \rho\left(u'(c_{k+1}) - 1\right), \quad k \geq 1,$$

which implies $u'(c_k) - 1 \geq \rho^{-k}\left(u'(c_1) - 1\right)$ with $u'(c_1) > 1$. In consequence, $\lim_{k\to\infty} u'(c_k) = \infty$ and $\lim_{k\to\infty} c_k = 0$. The budget constraint (14) implies $\lim_{k\to\infty} B_k = -\frac{\bar{l}}{1-q}$.

B.3 Case $D < \infty$

Since the default occurs at date D, we have $V_k \geq V_u^d(\underline{B})$ for $k = 1, \ldots, D-1$ and $V_{k0} \geq V_p^d(\underline{B})$ for $k = -1, 0, \ldots, D-1$. Assume that $V_{D-1,0} > V_p^d(\underline{B})$. We have from equations (15) and (22):

$$V_{D-1} = u\left(B_{D-2} + \bar{l} - qB_{D-1}\right) - \bar{l} + \beta\left((1-\rho)V_{D-1,0} + \rho V_u^d(\underline{B})\right),$$

with $V_{D-1} \geq V_u^d(\underline{B})$ and V_{D-1} decreasing in B_{D-1}. Therefore, the agent u unproductive for $D-1$ periods could increase her welfare by decreasing slightly B_{D-1}. This would not affect default incentives since $V_{D-1,0} > V_p^d(\underline{B})$, which by continuity still holds after a small decrease in B_{D-1}. This contradicts the optimality of agent's choices and this implies $V_{D-1,0} = V_p^d(\underline{B})$.

C Proof of Propositions 2

Let $D \geq 1$. Using equations (28)–(32) of Section A in Appendix, we have:

- n_0 goes up with D:
$$\frac{\partial n^p}{\partial \rho^{D-1}} = -\frac{(1-\theta)(1-\alpha)n^p}{\theta + (1-\theta)(1-\alpha)\rho^{D-1}} < 0.$$

- n_p^d goes down with D since the total population of p agents remains constant.

- $\sum_{k=1}^{D-1} n_k$ goes up with D:
$$\frac{\partial n^u}{\partial \rho^{D-1}} = -\frac{(1-\theta)(1-\alpha)n^p}{\theta + (1-\theta)(1-\alpha)\rho^{D-1}} - \frac{\theta}{\theta + (1-\theta)(1-\alpha)\rho^{D-1}} \frac{1-\alpha}{2-\alpha-\rho} < 0.$$

- n_u^d goes down with D since the total population of u agents remains constant.

Consider the planner program that can be rewritten as:

$$W = \max_{D \in \mathbb{N}_*} \max_{\substack{\{c_p^{d,D}, l_p^{d,D}, c_u^{d,D}\}, \\ \{l_{k0}^D\}_{-1 \le k < D}, \{c_k^D, B_k^D\}_{0 \le k < D} \in \mathcal{A}_D}} U + \beta W',$$

where $\mathcal{A}_D$ is the feasible set for a given $D \in \mathbb{N}_*$. We have added superscripts D to allocations in order to highlight the dependence on D. We have:

$$\begin{aligned} U = &\sum_{k=-1}^{D-1} n_{k0}^D \left(u\left(c_0^D\right) - l_{k0}^D\right) + \sum_{k=1}^{D-1} n_k^D \left(u\left(c_k^D\right) - \bar{l}\right) \\ &+ n_p^{d,D}\left(u\left(c_p^{d,D}\right) - l_p^d\right) + n_{u,1}^{d,D}\left(u\left(c_{u,1}^{d,D}\right) - \bar{l}\right) + \left(n_u^{d,D} - n_{u,1}^{d,D}\right)\left(u\left(c_u^{d,D}\right) - \bar{l}\right), \end{aligned}$$

where we also added the D superscript to population proportions. Let us show that $\mathcal{A}_D \subset \mathcal{A}_{D'}$ for any $D' \ge D$. To do so, we consider a feasible allocation that can be characterized by $(c_p^{d,D}, l_p^{d,D}, c_u^{d,D}, \{l_k^D\}_{-1 \le k < D}, \{c_{p,k}^D, B_k^D\}_{0 \le k < D}) \in \mathcal{A}_D$. Using budget constraint (1), we have:

$$B_{k-1}^{u,D} \ge c_k^{u,D} - \bar{l} + pB_k^{u,D}, 0 \le k < D-1, \tag{34}$$

$$B_{D-2}^{u,D} \ge c_{D-1}^{u,D} - \bar{l} + p(1-\rho) B_{D-1}^{u,D}, \tag{35}$$

$$c_{u,1}^{d,D} \le \varphi \bar{l} + \underline{B}, \tag{36}$$

$$c_u^{d,D} \le \varphi \bar{l}, \tag{37}$$

$$c_{p,k}^{p,D} + pB_0^{p,D} \le l_k^{p,D} + B_k^{p,D}, 0 \le k < D, \tag{38}$$

$$c_p^{d,D} \le \varphi l_p^{d,D}. \tag{39}$$

From the previous remark about the impact of D on population shares, we know that the population of p and u agents who are excluded is shrinking, while the population of p and u

countries that participate is going up. We define for unproductive countries:

$$B_k^{D'} = \begin{cases} B_k^D & \text{for } n_k^D \text{ countries} \\ 0 & \text{for } n_k^{D'} - n_k^D \text{ countries} \end{cases},$$

$$c_k^{D'} = \begin{cases} c_{p,k}^D & \text{for } n_k^D \text{ countries} \\ c_{p,k}^d & \text{for } n_k^{D'} - n_k^D \text{ countries} \end{cases},$$

$$l_k^{D'} = \begin{cases} \bar{l} & \text{for } n_k^D \text{ countries} \\ \varphi \bar{l} & \text{for } n_k^{D'} - n_k^D \text{ countries} \end{cases},$$

$$\begin{aligned} c_u^{d,D'} &= c_u^{d,D} \text{ for } n_u^{d,D'} - n_{u,1}^{d,D'} \text{ countries}, \\ c_{u,1}^{d,D'} &= c_{u,1}^{d,D} \text{ for } n_{u,1}^{d,D'} \text{ countries}, \\ l_u^{d,D'} &= \varphi \bar{l} \text{ for } n_u^{d,D'} \text{ countries}, \end{aligned}$$

where the idea is to "view" some excluded agents for D as participating agents for D', which is possible since we have (i) $n_k^D \leq n_k^{D'}$, (ii) $n_u^{d,D'} \leq n_u^{d,D}$, and (iii) $n_k^D + n_u^{d,D} = n_k^{D'} + n_u^{d,D'}$. It is obvious that constraint (36) holds for D', and that (34) and (35) also hold for D' (for a population of n_k^D agents that were in the same state for D). For the remaining population of $n_k^{D'} - n_k^D$ agents, equation (34) for D' becomes $c_u^{d,D'} - \bar{l} = c_u^{d,D} - \bar{l} + 0 = (\varphi - 1)\bar{l} \leq 0$, which therefore holds. By the same token, equation (35) for D' also holds.

For productive agents, we similarly define:

$$B_0^{D'} = \begin{cases} B_0^D & \text{for } n_0^D \text{ countries} \\ 0 & \text{for } n_0^{D'} - n_0^D \text{ countries} \end{cases},$$

$$c_0^{D'} = \begin{cases} c_0^D & \text{for } n_0^D \text{ countries} \\ c_p^{d,D} & \text{for } n_0^{D'} - n_0^D \text{ countries} \end{cases},$$

$$l_{k0}^{D'} = \begin{cases} l_{k0}^{p,D} & \text{for } n_{0k}^D \text{ countries} \\ \varphi l_p^{d,D} & \text{for } n_{0k}^{D'} - n_{0k}^D \text{ countries} \end{cases},$$

$$\begin{aligned} c_p^{d,D'} &= c_p^{d,D} \text{ for } n_p^{d,D'} \text{ countries}, \\ l_p^{d,D'} &= \varphi l_p^{d,D} \text{ for } n_p^{d,D'} \text{ countries}, \end{aligned}$$

Note that equation (39) holds for D' by construction. Regarding equation (38), we have $c_p^{d,D} \leq \varphi l_p^{d,D} \leq l_p^{d,D} \leq l_p^{d,D'}$, which therefore holds.

In consequence the allocation $\left(c_p^{d,D}, l_p^{d,D}, c_{u,1}^{d,D}, c_u^{d,D}, \{l_k^D\}_{-1\leq k<D}, \{c_k^D, B_k^D\}_{0\leq k<D}\right)$ belongs to the feasible set $\mathcal{A}_{D'}$. We deduce that $\mathcal{A}_D \subset \mathcal{A}_{D'}$. This has two consequences.

1. For a given price, feasible sets are increasing in D (in the sense of inclusion). This implies that any welfare level for D can be reached for any $D' \geq D$. The welfare is therefore increasing in D.

2. By the same token, we can therefore rank competitive equilibria by an aggregate welfare criterion using D. This proves Proposition 2.

D Formal derivation of the quasi-planner's allocation

We use the same notation as in the market economy of Section 2, except that we add a tilde for the quasi-planner allocation.[15] First, the planner chooses the optimal default decision $\tilde{D}$. The quasi-planner then chooses the consumption $\tilde{c}_0$, net savings $\tilde{B}_0$, and labor supply $\tilde{l}_{k0}$ of productive countries that have been unproductive for $k = -1, 0, \ldots, \tilde{D}$ consecutive periods. The planner also chooses the consumption $\tilde{c}_k$ and the savings $\tilde{B}_k$ of unproductive countries for $k = 1, 2, \ldots, \tilde{D}$ consecutive periods. As before, the net wealth of countries reentering the financial markets is zero, $\tilde{B}_{-1} = 0$. The quasi-planner is concerned with the instantaneous aggregate utility $\tilde{U}$ expressed as:

$$\tilde{U} = \sum_{k=-1}^{\tilde{D}-1} \tilde{n}_{k0} \left(u\left(\tilde{c}_0\right) - \tilde{l}_{k0}\right) + \sum_{k=1}^{\tilde{D}-1} \tilde{n}_k \left(u\left(\tilde{c}_k\right) - \bar{l}\right) \tag{40}$$
$$+ \tilde{n}_p^d \left(u\left(\tilde{c}_p^d\right) - \tilde{l}_p^d\right) + \tilde{n}_{u,1}^d \left(u\left(\tilde{c}_{u,1}^d\right) - \bar{l}\right) + \left(\tilde{n}_u^d - \tilde{n}_{u,1}^d\right)\left(u\left(\tilde{c}_u^d\right) - \bar{l}\right),$$

where the tilde in population shares highlights their dependence in $\tilde{D}$. In equation (40), we distinguish unemployed countries that have just defaulted and reimburse $\underline{B}$. Formal expressions of population shares have the same expression as in the market economy, given in Appendix A.

[15] To simplify the exposition, we directly assume that productive countries consume the same amount, which is a direct outcome of the labor choice made in those countries by the quasi-planner.

The quasi-planner's program can be expressed recursively as follows:

$$\tilde{W} = \max_{\substack{\{\tilde{q},\tilde{D},\tilde{c}_p^d,\tilde{l}_p^d,\tilde{c}_u^d\},\\ \{\tilde{l}_{k0}\}_{-1\leq k<\tilde{D}-1},\{\tilde{c}_k\}_{1\leq k<\tilde{D}-1},\{\tilde{B}_k\}_{0\leq k<\tilde{D}-1}}} \tilde{U} + \beta \tilde{W}', \tag{41}$$

subject to the set of country budget constraints, which are similar to (14)–(16) in the market economy:

$$\tilde{B}_{k-1} = \tilde{c}_k - \bar{l} + \tilde{q}\tilde{B}_k, \quad 1 \leq k < \tilde{D} - 1, \tag{42}$$

$$\tilde{B}_{\tilde{D}-2} = \tilde{c}_{D-1} - \bar{l} + \tilde{q}\,(1-\rho)\,\tilde{B}_{\tilde{D}-1} + \tilde{q}\rho\underline{B}, \tag{43}$$

$$\tilde{c}_0 + \tilde{q}\tilde{B}_0 = \tilde{l}_{k0} + \tilde{B}_k, \quad -1 \leq k \leq \tilde{D} - 1, \tag{44}$$

$$\tilde{c}_p^d = \varphi \tilde{l}_p^d,\ \tilde{c}_{u,1}^d = \varphi \bar{l} + \underline{B},\ \tilde{c}_u^d = \varphi \bar{l}, \tag{45}$$

and subject to individual participation constraints (similar to (17) and (19) in the market economy):

$$\tilde{V}_{k0} \geq \tilde{V}_p^d\left(\underline{B}\right), \quad -1 \leq k < D, \tag{46}$$

$$\tilde{V}_k \geq \tilde{V}_u^d\left(\underline{B}\right), \quad 1 \leq k < D. \tag{47}$$

The quantities $\tilde{V}_{k0}$ and $\tilde{V}_k$ represent the intertemporal welfare of different country types, with expressions similar to (21) and (22) for the market economy. The last constraint of the quasi-planner program is the financial market clearing:

$$\sum_{k=0}^{\tilde{D}-2} \tilde{n}_k \tilde{B}_k + (1-\rho)\tilde{n}_{\tilde{D}-1}\tilde{B}_{\tilde{D}-1} + \rho \tilde{n}_{\tilde{D}-1}\underline{B} = 0. \tag{48}$$

The key difference between the market economy and the constrained-efficient allocation is that the quasi-planner internalizes the effect of saving and default decisions on the price of the safe asset, and thus on aggregate risk-sharing.

The solution of the quasi-planner program is straightforward. We now present the corresponding first-order conditions in order to provide insights into the constrained-efficient allocation. We denote by λ the Lagrange multiplier of the financial market clearing constraint (48), which corresponds to the social value for the planner of one additional unit of the world safe asset. Euler equations can be written as:

$$\tilde{q} + \lambda = \beta \left(\alpha + (1-\alpha) u'(\tilde{c}_1)\right), \tag{49}$$

$$\tilde{q} u'(\tilde{c}_k) + \lambda = \beta \left(1 - \rho + \rho u'(\tilde{c}_{k+1})\right), \quad 1 \leq k \leq \tilde{D} - 2, \tag{50}$$

$$q\tilde{u'}\left(c_{\tilde{D}-1}\right) + \lambda \geq \beta. \tag{51}$$

The price is determined by the first-order condition:

$$\sum_{k=-1}^{D-1} n_k^p B_k u'(c_k^p) + \sum_{k=1}^{D-2} n_k^u B_k^u u'(c_k^u) + \left((1-\rho) n_{D-1}^u B_{D-1}^u + \rho n_{D-1}^u \underline{B}\right) u'(c_{D-1}^u) = 0. \tag{52}$$

Before commenting on these expressions, it is worth noting that when $\lambda = 0$, i.e. when there is no internalization of the effect of saving on prices, we find the same first-order conditions as in the market economy. The difference between the two allocations therefore stems solely from the internalization of the pecuniary externality.

The internalization of the pecuniary externality affects the Euler equations through the additional net gain of saving, which is captured by λ. Furthermore, the quasi-planner's price setting implies an additional first-order condition, which is equation (52). This equation stipulates that the quasi-planner sets the price of the safe asset such that the redistributive effects maximize welfare across buyers and sellers.

E Proof of Proposition 3

We show that the quasi-planner allocation is either autarky or it is characterized by $\tilde{q} + \lambda = \beta$ and $u'(\tilde{c}_k) = 1$ for $k = 0, \ldots, \tilde{D} - 1$.

As a preliminary, note that for some parameter values, the only equilibrium is the no-trade equilibrium characterized by $\tilde{B}_k = 0$ for all k. Following the results of Bulow and Rogoff (1989), this can be the case if φ (the output cost of default) is close to 1. In what follows, we assume that it is not the case and that there is a k such that $B_k > 0$. We proceed in three steps.

Step 1. If $\tilde{q} + \lambda = \beta$, equations (49)–(51) imply $u'(\tilde{c}_1) = 1$, and then $u'(\tilde{c}_k) = 1$ for $k = 1, \ldots, \tilde{D} - 1$. Condition (52) holds (as in any equilibrium).

Step 2. We show that there is no constrained-efficient equilibrium where $\tilde{q}+\lambda>\beta$. We proceed by contradiction. Assume that $\tilde{q}+\lambda>\beta$. First, from equation (49), we obtain $u'(\tilde{c}_k)>1$. Then using (50), we recursively show that $u'(\tilde{c}_{k+1})>u'(\tilde{c}_k)$, for $k=0,\ldots,\tilde{D}-1$ and $(\tilde{c}_k)_k$ is decreasing. Second, budget constraints (42) then imply that $(B_k)_k$ is decreasing.

We now show that the financial market clearing condition (48) and the first-order condition (52) cannot hold at the same time. Let us define k_0 as the index such that $B_{k_0}\geq 0>B_{k_0+1}$. The index k_0 exists because $(B_k)_k$ is decreasing and equality (48) holds. This implies $\tilde{B}_k\geq 0$ for all $k=0,\ldots,k_0$ and $\tilde{B}_k<0$ for $k=k_0+1,\ldots,\tilde{D}-2$. Since $(\tilde{c}_k)_k$ is decreasing, we deduce:

$$\sum_{k=0}^{k_0}\tilde{n}_k\tilde{B}_k u'(\tilde{c}_k)<u'(\tilde{c}_{k_0})\sum_{k=0}^{k_0}\tilde{n}_k\tilde{B}_k, \tag{53}$$

as well as:

$$\begin{aligned}-\left(\sum_{k=k_0+1}^{\tilde{D}-2}\tilde{n}_k\tilde{B}_k u'(\tilde{c}_k)+\Big((1-\rho)\tilde{n}_{\tilde{D}-1}\tilde{B}_{\tilde{D}-1}+\rho\tilde{n}_{\tilde{D}-1}\underline{B}\Big)u'(\tilde{c}_{D-1})\right)> \\ -u'(\tilde{c}_{k_0+1})\left(\sum_{k=k_0+1}^{\tilde{D}-2}\tilde{n}_k\tilde{B}_k+(1-\rho)\tilde{n}_{\tilde{D}-1}\tilde{B}_{\tilde{D}-1}+\rho\tilde{n}_{\tilde{D}-1}\underline{B}\right).\end{aligned} \tag{54}$$

Furthermore, the financial market clearing condition (48) implies:

$$\sum_{k=0}^{k_0}\tilde{n}_k\tilde{B}_k=-\left(\sum_{k=k_0+1}^{\tilde{D}-2}\tilde{n}_k\tilde{B}_k+(1-\rho)\tilde{n}_{\tilde{D}-1}\tilde{B}_{\tilde{D}-1}+\rho\tilde{n}_{\tilde{D}-1}\underline{B}\right),$$

with which equations (53) and (54) implies:

$$-\left(\sum_{k=k_0+1}^{\tilde{D}-2}\tilde{n}_k\tilde{B}_k u'(\tilde{c}_k)+\Big((1-\rho)\tilde{n}_{\tilde{D}-1}\tilde{B}_{\tilde{D}-1}+\rho\tilde{n}_{\tilde{D}-1}\underline{B}\Big)u'(\tilde{c}_{D-1})\right)>\sum_{k=0}^{k_0}\tilde{n}_k\tilde{B}_k u'(\tilde{c}_k).$$

The last inequality shows that the condition (52) cannot hold, which is a contradiction.

Step 3. We consider the case $\tilde{q}+\lambda<\beta$ and show that no constrained-efficient equilibrium exists. The proof is analogous to the one in the second step, except that $(\tilde{c}_k)_k$ and $(\tilde{B}_k)_k$ are shown to be increasing (instead of decreasing). We still obtain a contradiction by showing that the financial clearing (48) and the first-order condition (52) cannot simultaneously hold.

F Proof of Proposition 4

F.1 Full risk-sharing implementation

We now provide the amount of liquidity implementing full risk sharing and finite default horison D. Existence conditions are then discussed. An equilibrium in the IFI economy is thus a default horizon D, an IFI policy (F, τ), an asset price q, allocations $c_p^d, l_p^d, c_u^d, \{l_k\}_{-1 \leq k < D}, \{c_k, B_k\}_{0 \leq k < D}$, such that: 1) for a given q and (F, τ), allocations are consistent with individual country programs; 2) the financial market clears and (27) holds; 3) the IFI policy is balanced and (26) holds. It is straightforward to show that Proposition 1 holds, as the proof does not depend on contributions.

Then full risk-sharing is implemented in the IFI economy when $q = \beta$. Indeed, as $q = \beta$, the first-order conditions, identical to (11)–(13), for $k = 0, \ldots, D-1$ yield $c_0 = c_k = u'^{-1}(1) \equiv c$. Using the tractability in our framework, we exhibit the 3 equations determining the size of the IFI for any finite D.

First equation. The budget constraints of unproductive countries, now including the payment τ, lead after backward iteration to:

$$B_k = \frac{B_0}{\beta^k} + (c - \bar{l} + \tau)\frac{1 - \beta^{-k}}{1 - \beta}, \tag{55}$$

where B_0 is the saving of participating productive countries. The welfare of participating unproductive countries is:

$$V_k = u(c) - \bar{l} + \beta(1-\rho)V_p^c(0) + \beta(1-\rho)B_k + \beta\rho V_{k+1},$$

which yields after forward iteration:

$$\begin{aligned} V_1 = &\Big(u(c) - \bar{l} + \beta(1-\rho)V_p^c(0)\Big)\,\frac{1 - (\beta\rho)^{D-1}}{1 - \beta\rho} \\ &+ (c - \bar{l} + \tau)\frac{\beta(1-\rho)}{1-\beta}\frac{1 - (\beta\rho)^{D-1}}{1 - \beta\rho} \\ &+ \left(B_0 - \frac{c - \bar{l} + \tau}{1 - \beta}\right)(1 - \rho^{D-1}) + (\beta\rho)^{D-1}\, V_u^d(\overline{B}). \end{aligned} \tag{56}$$

The welfare of a productive participating country with a wealth 0 in the IFI economy is:

$$V_p^c(0) = u(c) - (c + \tau + \beta B_0) + \beta\alpha V_p^c(0) + \beta\alpha B_0 + \beta(1-\alpha)V_1,$$

or using the expression (56) of V_1:

$$
\begin{aligned}
(1-\beta\alpha)\,V_p^c(0) = u(c) - (c+\tau+\beta B_0) + \beta\alpha B_0 \\
+\left(u(c)-\bar{l}+\beta(1-\rho)V_p^c(0)\right)\beta(1-\alpha)\frac{1-(\beta\rho)^{D-1}}{1-\beta\rho} \\
+(c-\bar{l}+\tau)\frac{\beta^2(1-\alpha)(1-\rho)}{1-\beta}\frac{1-(\beta\rho)^{D-1}}{1-\beta\rho} \\
+\beta(1-\alpha)\left(B_0-\frac{c-\bar{l}+\tau}{1-\beta}\right)(1-\rho^{D-1})+(\beta\rho)^{D-1}\beta(1-\alpha)V_u^d(\overline{B}).
\end{aligned}
\tag{57}
$$

As $V_p^c(B_{D-1}) = V_p^d(\overline{B})$, we have $V_p^c(0) = V_p^d(\underline{B}) - B_{D-1}$ or using (55):

$$
V_p^c(0) = V_p^d(\underline{B}) - \beta^{-(D-1)}B_0 - \frac{1-\beta^{-(D-1)}}{1-\beta}(c-\bar{l}+\tau). \tag{58}
$$

Substituting (58) into (57), we obtain:

$$
\Psi_1\tau = \Psi_2 - \Psi_3 B_0, \tag{59}
$$

with:

$$
\begin{aligned}
\Psi_1 &= \beta^2(1-\alpha)(1-\rho)\frac{1-(\beta\rho)^{D-1}}{1-\beta\rho}\frac{\beta^{-(D-1)}}{1-\beta} \\
&\quad+\frac{1}{1-\beta}\left(\rho^{D-1}\beta(1-\alpha)-\beta^{-(D-1)}(1-\beta\alpha)\right), \\
\Psi_2 &= -\left(u(c)-\bar{l}\right)\left(1+\beta(1-\alpha)\frac{1-(\beta\rho)^{D-1}}{1-\beta\rho}\right) \\
&\quad-(c-\bar{l})\left(\beta^2(1-\alpha)(1-\rho)\frac{1-(\beta\rho)^{D-1}}{1-\beta\rho}\frac{\beta^{-(D-1)}}{1-\beta}\right. \\
&\qquad\left.+\frac{1}{1-\beta}\left(\rho^{D-1}\beta(1-\alpha)-\beta^{-(D-1)}(1-\beta\alpha)\right)\right) \\
&\quad\left(1-\beta\alpha-\beta^2(1-\alpha)(1-\rho)\frac{1-(\beta\rho)^{D-1}}{1-\beta\rho}\right)V_p^d(\underline{B})-(\beta\rho)^{D-1}\beta(1-\alpha)V_u^d(\underline{B}), \\
\Psi_3 &= \left(1-\beta\alpha-\beta(1-\alpha)\frac{\beta(1-\rho)+(1-\beta)(\beta\rho)^{D-1}}{1-\beta\rho}\right)\beta^{-(D-1)}.
\end{aligned}
$$

The two value functions $V_p^d(\underline{B})$ and $V_u^d(\underline{B})$ are endogenous values, as the probability to reenter is positive. The functions can be expressed as a function of $V_p^c(0)$, which is the value of a country reentering the market. Using (57), they can be written as a function of B_0 and τ.

Second equation. Using equations (55) and (29), the IFI clearing condition becomes:

$$\frac{F}{(1-\alpha)n_0} = \frac{B_0}{1-\alpha} + \sum_{k=1}^{D-1} \rho^{k-1} \left(\frac{B_0}{\beta^k} + (c - \bar{l} + \tau)\frac{1-\beta^{-k}}{1-\beta} \right),$$

$$= B_0 \left(\frac{1}{1-\alpha} + \frac{1-\rho^{D-1}\beta^{-(D-1)}}{\beta-\rho} \right) + \frac{c-\bar{l}+\tau}{1-\beta} \left(\frac{1-\rho^{D-1}}{1-\rho} - \frac{1-\rho^{D-1}\beta^{-(D-1)}}{\beta-\rho} \right). \quad (60)$$

Third equation. The third equation is the budget constraint of the IFI (26):

$$(1-\beta)F = n_p\tau. \quad (61)$$

The three linear equations (59), (60), and (61) form a linear system in three unknowns in B_0, F, τ. Except particular cases (which would correspond to a non-invertible matrix for the linear system – which is a zero-measure set), there is always a solution. These three equations ensure that there exists an IFI economy equilibrium with a finite F^*, and $q = \beta$.

Existence conditions. Note that if productive countries are saving B_0 when belonging to the IFI risk-sharing arrangement, then the optimal saving decision when unproductive is given by the sequence B_k satisfying Euler conditions and provided above. Assumption 4. in Section 6.1 ensures that the IFI can impose a maximum amount of saving B_0 (whereas productive countries would like to save more). First, the equilibrium construction ensures that the utility of participating countries is higher that the one if they default, which determines the optimal default horizon D. Second, the equilibrium construction ensures that the the inter-temporal welfare of productive participating countries is higher that the one in default, as $V_p^c(B_{D-1}) = V_p^d(\overline{B})$.

The last participating condition must ensure that countries accept to join the IFI risk-sharing arrangement. As productive countries get out of the default state with 0 wealth, this can be written as $V_p^c(0) > V_p^d(0)$.

F.2 Existence conditions with $q > \beta$

For $q > \beta$, the amount of liquidity chosen by a productive participating country B_0 and the equilibrium outcome are determined by the same equations as in the market economy. The equilibrium construction of the default horizon D ensures that all participating countries are better-off in the IFI risk-sharing arrangement. The additional participation constraint can be thus written as $V_p^c(0) > V_p^d(0)$. It ensures that countries out of the default state join the IFI risk-sharing arrangement when all countries not in the default state participate in this arrangement.

G The Hegemon economy

We here follow Farhi and Maggiori (2018) and introduce a country called the Hegemon into the previous economy. This country has a positive mass and internalizes the impact of its choices on the asset price. More precisely, we consider the case where a country is larger than other countries and wields market power on the world financial market. This extension aims to show that the main results of our analysis are robust to the relaxation of the assumption of perfectly symmetric countries. In particular, even in this environment, excess default remains at the equilibrium, and an international liquidity provider can restore constrained-efficiency. We now assume that the world includes, in addition to the continuum of countries presented above, a country referred to as the Hegemon, of mass m with financial market power. The Hegemon has the same preferences as any other country. Clearing on the world financial market can now be expressed as follows:

$$mB^H + \int_{i \in I} q\left(B'^i, s^i\right) B'^i G\left(di\right) = 0, \tag{62}$$

where B^H denotes the net borrowing demand of the Hegemon. Other notation remains unchanged.

As the Hegemon has a positive mass, it internalizes the effect of its (net) borrowing demand B^H on the price of the world safe asset $q\left(B^H\right)$:

$$V^H\left(B^H, X\right) = \max_{\{c^H, B^{H\prime}\}} u\left(c^H\right) + \beta V^H\left(B^{H\prime}, X'\right), \tag{63}$$

$$\text{s.t. } c^H = y^H + B^H - q\left(B^{H\prime}\right) B^{H\prime}. \tag{64}$$

The asset choice of the Hegemon is derived by the solution of the problem (63)–(64). Even though the function $B \mapsto q\left(B\right)$ may be discontinuous, it can be shown to be globally decreasing. Furthermore, we can use the results of Clausen and Strub (2019) (see in particular their section 5.1) to show that the optimum is characterized by the first-order condition $q(B^H) + q'\left(B^{H\prime}\right) B^{H\prime} = \beta$, which implies that the equilibrium price is strictly above β.

The quasi-planner aims at maximizing the instantaneous aggregate utility $\tilde{U}$ expressed as:

$$\tilde{U} = mu\left(\tilde{c}^H\right) + \sum_{k=-1}^{\tilde{D}-1} \tilde{n}_{k0}\left(u\left(\tilde{c}_0\right) - \tilde{l}_{k0}\right) + \sum_{k=1}^{\tilde{D}-1} \tilde{n}_k\left(u\left(\tilde{c}_k\right) - \bar{l}\right) \tag{65}$$
$$+ \tilde{n}_p^d\left(u\left(\tilde{c}_p^d\right) - \tilde{l}_p^d\right) + \tilde{n}_u^d\left(u\left(\tilde{c}_u^d\right) - \bar{l}\right),$$

where the tilde on variables highlights their dependence in $\tilde{D}$. The following proposition characterizes the constrained-efficient equilibrium.

Proposition 5 (Constrained-efficient allocation) *Either the constrained-efficient allocation is autarky or it is characterized by:*

- *the asset price is equal to* $\tilde{q} = \beta$*;*
- *there is full insurance:* $\tilde{c}_k = \tilde{c}^H = u'^{-1}(1)$ *for* $k = 0, \ldots, \tilde{D} - 1$*;*
- *the net position* $\tilde{B}^H$ *of the Hegemon is determined by the sign of* $u'^{-1}(1) - y^H$*. If it is positive and* $u'^{-1}(1) \geq y^H$*, then the Hegemon is a net lender, and if the opposite holds, then the Hegemon is a net borrower.*

The proof is the following. In the Hegemon economy, the quasi-planner program can be expressed recursively as follows:

$$\tilde{W} = \max_{\substack{\{\tilde{q},\tilde{D},\tilde{c}_p^d,\tilde{l}_p^d,\tilde{c}_u^d,\tilde{c}^H,\tilde{l}^H\},\\ \{\tilde{l}_{k0}\}_{-1\leq k<\tilde{D}-1},\{\tilde{c}_k\}_{1\leq k<\tilde{D}-1},\{\tilde{B}_k\}_{0\leq k<\tilde{D}-1},\tilde{B}^{H\prime}}} \tilde{U} + \beta\tilde{W}', \tag{66}$$

subject to the set of country budget constraints (similar to (14)–(16) in the market economy):

$$\tilde{B}_{k-1} = \tilde{c}_k - \bar{l} + \tilde{q}\tilde{B}_k, \quad 1 \leq k < \tilde{D} - 1, \tag{67}$$

$$\tilde{B}_{\tilde{D}-2} = \tilde{c}_{D-1} - \bar{l} + \tilde{q}\left(1-\rho\right)\tilde{B}_{\tilde{D}-1} + \tilde{q}\rho\underline{B}, \tag{68}$$

$$\tilde{c}_0 + \tilde{q}\tilde{B}_0 = \tilde{l}_{k0} + \tilde{B}_k, \quad -1 \leq k \leq \tilde{D} - 1, \tag{69}$$

$$\tilde{c}_p^d = \varphi\tilde{l}_p^d, \text{ and } \tilde{c}_u^d = \varphi\bar{l}, \tag{70}$$

$$\tilde{c}^H = y^H + \tilde{B}^H - \tilde{q}\tilde{B}^{H\prime}. \tag{71}$$

and subject to participation constraints (similar to (17) and (19) in the market economy): $\tilde{V}_{k0} \geq \tilde{V}_p^d(\underline{B}), \quad -1 \leq k < D$ and $\tilde{V}_k \geq \tilde{V}_u^d(\underline{B}), \quad 1 \leq k < D$. The planner understands the effect of the saving decisions of the Hegemon and of all other countries on the safe asset price.

$$\sum_{k=0}^{\tilde{D}-2} \tilde{n}_k \tilde{B}_k + (1-\rho)\tilde{n}_{\tilde{D}-1}\tilde{B}_{\tilde{D}-1} + \rho\tilde{n}_{\tilde{D}-1}\underline{B} + m\tilde{B}^{H'} = 0. \tag{72}$$

Euler equations can be written as:

$$\tilde{q} + \lambda = \beta\left(\alpha + (1-\alpha)\, u'\left(\tilde{c}_1\right)\right), \tag{73}$$

$$\tilde{q}u'\left(\tilde{c}_k\right) + \lambda = \beta\left(1 - \rho + \rho u'\left(\tilde{c}_{k+1}\right)\right), \quad 1 \leq k \leq \tilde{D} - 2, \tag{74}$$

$$q\tilde{u}'\left(c_{\tilde{D}-1}\right) + \lambda \geq \beta, \text{ and } \tilde{q} + \lambda = \beta. \tag{75}$$

The price is determined by the following first-order condition:

$$0 = mu'\left(\tilde{c}^H\right) B^{H'} + \sum_{k=-1}^{D-1} n_k^p B_k u'\left(c_k^p\right) + \sum_{k=1}^{D-2} n_k^u B_k^u u'\left(c_k^u\right) \tag{76}$$

$$+ n_{D-1}^u\left((1-\rho)B_{D-1}^u + \rho\underline{B}\right) u'\left(c_{D-1}^u\right).$$

An equilibrium is $u'(\tilde{c}^H) = u'(c_k^p) = u'(c_k^u) = u'(c_{D-1}^u) = 1$. We obtain: $u'^{-1}(1) = y^H + (1-\tilde{q})\,\tilde{B}^H$ and $\tilde{B}^H = -\frac{y^H - u'^{-1}(1)}{1-\beta}$, which concludes the proof.

The novel part of the Proposition consists in the characterization of the net position of the Hegemon. The budget constraint (64) of the Hegemon coupled with the two first points of Proposition 5 imply that the net borrowing position of the Hegemon is defined by $\tilde{B}^H = \frac{u'^{-1}(1) - y^H}{1-\beta}$, thereby proving the last point. Another take-away of Proposition 5 is that when the Hegemon internalizes the effect of its choices on the asset price, then the market allocation is not constrained-efficient.

The asset price at the market equilibrium is higher and the equilibrium features imperfect insurance. Defaults at the equilibrium are too high because there are insufficient safe assets in the economy. In other words, the result found for the symmetric equilibrium, where there is excess default at the equilibrium, still holds in the presence of a Hegemon with financial market power. As in the symmetric economy, an IFI providing liquidity to all countries can increase the aggregate welfare. The IFI turns out to be a net liquidity provider, which induces a fall in the price of liquidity and neutralizes the market power of the Hegemon.